Joe Niemela

Cryogenic Turbulence

K.R. Sreenivasan
The Abdus Salam International Centre for Theoretical Physics

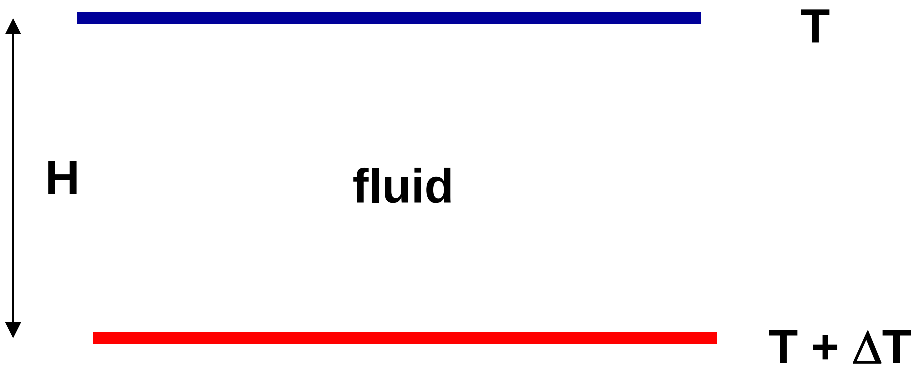
International Symposium on Contemporary Physics
National Centre for Physics
Islamabad

Greg Bewley

March 27, 2007

Dan Lathrop

Heat transport through bounded fluid



f ≡ Nusselt number, Nu

Ra ≡ Rayleigh number
$\alpha g \Delta T H^3/[\nu\kappa]$

For small ΔT (no fluid motion): Fourier's law holds and heat transport = κΔT/H

$Nu = Ra^0 = 1$

For moderately large ΔT (fluid motion smooth) heat transport = κΔT/H × f(other parameters)

$Nu = C_1 Ra^{1/4}$

For large ΔT, the fluid motion is turbulent (temporally and spatially complex), what is the heat transport law?

$Nu = C_2 Ra^{\beta}$?

No full-fledged theory!

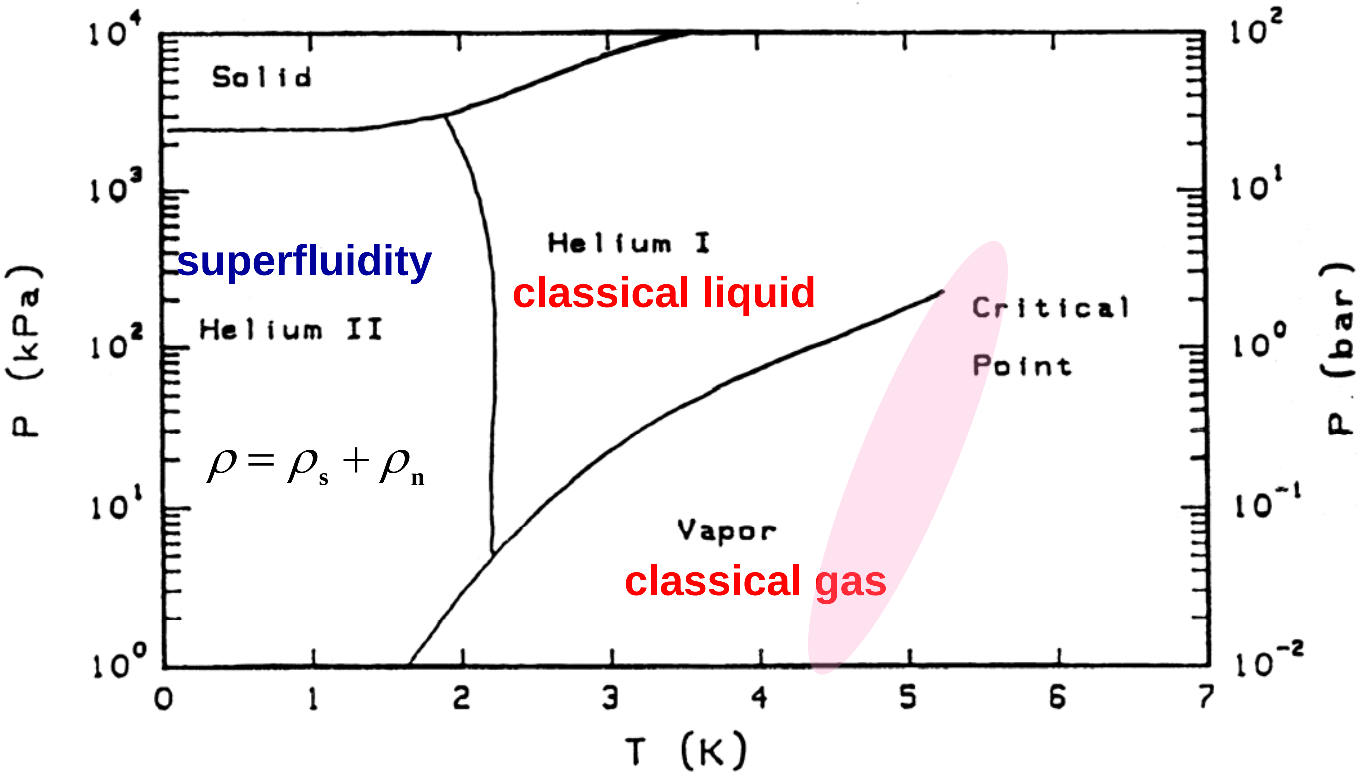
How large an Ra should interest us?

1. Human body: Ra ~ 10^{8}
2. Atmosphere: Ra ~ 10^{13}
3. Ocean convection: Ra ~ 10^{20}
4. Stellar convection: Ra ~ 10^{22}

The challenge

Experimentally, vary Ra by 14 orders or so and reach ~ 10^{22}.

$$\mathbf{Ra} = (\alpha/\nu\kappa)\mathbf{g}\Delta\mathbf{TH}^{3}$$



$$\rho = \rho_s + \rho_n$$

Helium I: $\nu = 2 \times 10^{-8}$ m²/s (water: 10^{-6} m²/s, air: 1.5×10^{-5} m²/s)

$$Ra = g \cdot \left(\frac{\alpha}{\nu\kappa} \right) \cdot \Delta T \cdot H^3$$

4.4 K, 2 mbar: $\alpha/\nu\kappa = 6.5 \times 10^9$

5.25 K, 2.4 bar: $\alpha/\nu\kappa = 5.8 \times 10^{-3}$

Letters to the Editor

The Editor does not hold himself responsible for opinions expressed by his correspondents. He cannot undertake to return, or to correspond with the writers of, rejected manuscripts intended for this or any other part of NATURE. *No notice is taken of anonymous communications.*

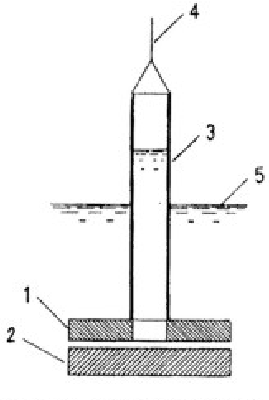
NOTES ON POINTS IN SOME OF THIS WEEK'S LETTERS APPEAR ON P. 83.

CORRESPONDENTS ARE INVITED TO ATTACH SIMILAR SUMMARIES TO THEIR COMMUNICATIONS.

Viscosity of Liquid Helium below the λ-Point

THE abnormally high heat conductivity of helium II below the λ-point, as first observed by Keesom, suggested to me the possibility of an explanation in terms of convection currents. This explanation would require helium II to have an abnormally low viscosity; at present, the only viscosity measurements on liquid helium have been made in Toronto[1], and showed that there is a drop in viscosity below the λ-point by a factor of 3 compared with liquid helium at normal pressure, and by a factor of 8 compared with the value just above the λ-point. In these experiments, however, no check was made to ensure that the motion was laminar, and not turbulent.

The important fact that liquid helium has a specific density ρ of about 0·15, not very different from that of an ordinary fluid, while its viscosity μ is very small comparable to that of a gas, makes its kinematic viscosity $\nu=\mu/\rho$ extraordinary small. Consequently when the liquid is in motion in an ordinary viscosimeter, the Reynolds number may become very high, while in order to keep the motion laminar, especially in the method used in Toronto, namely, the damping of an oscillating cylinder, the Reynolds number must be kept very low. This requirement was not fulfilled in the Toronto experiments, and the deduced value of viscosity thus refers to turbulent motion, and consequently may be higher by any amount than the real value.



The very small kinematic viscosity of liquid helium II thus makes it difficult to measure the viscosity. In an attempt to get laminar motion the following method (shown diagramatically in the accompanying illustration) was devised. The viscosity was measured by the pressure drop when the liquid flows through the gap between the disks 1 and 2; these disks were of glass and were optically flat, the gap between them being adjustable by mica distance pieces. The upper disk, 1, was 3 cm. in diameter with a central hole of 1·5 cm. diameter, over which a glass tube (3) was fixed. Lowering and raising this plunger in the liquid helium by means of the thread (4), the level of the liquid column in the tube 3 could be set above or below the level (5) of the liquid in the surrounding Dewar flask. The amount of flow and the pressure were deduced from the difference of the two levels, which was measured by cathetometer.

The results of the measurements were rather striking. When there were no distance pieces between the disks, and the plates 1 and 2 were brought into contact (by observation of optical fringes, their separation was estimated to be about half a micron), the flow of liquid above the λ-point could be only just detected over several minutes, while below the λ-point the liquid helium flowed quite easily, and the level in the tube 3 settled down in a few seconds. From the measurements we can conclude that the viscosity of helium II is at least 1,500 times smaller than that of helium I at normal pressure.

The experiments also showed that in the case of helium II, the pressure drop across the gap was proportional to the square of the velocity of flow, which means that the flow must have been turbulent. If, however, we calculate the viscosity, assuming the flow to have been laminar, we obtain a value of the order 10^{-9} c.g.s., which is evidently still only an upper limit to the true value. Using this estimate, the Reynolds number, even with such a small gap, comes out higher than 50,000, a value for which turbulence might indeed be expected.

We are making experiments in the hope of still further reducing the upper limit to the viscosity of liquid helium II, but the present upper limit (namely, 10^{-9} c.g.s.) is already very striking, since it is more than 10^4 times smaller than that of hydrogen gas (previously thought to be the fluid of least viscosity). The present limit is perhaps sufficient to suggest, by analogy with supraconductors, that the helium below the λ-point enters a special state which might be called a 'superfluid'.

As we have already mentioned, an abnormally low viscosity such as indicated by our experiments might indeed provide an explanation for the high thermal conductivity, and for the other anomalous properties observed by Allen, Peierls, and Uddin[2]. It is evidently possible that the turbulent motion, inevitably set up in the technical manipulation required in working with the liquid helium II, might on account of the great fluidity, not die out, even in the small capillary tubes in which the thermal conductivity was measured; such turbulence would transport heat extremely efficiently by convection.

P. KAPITZA.

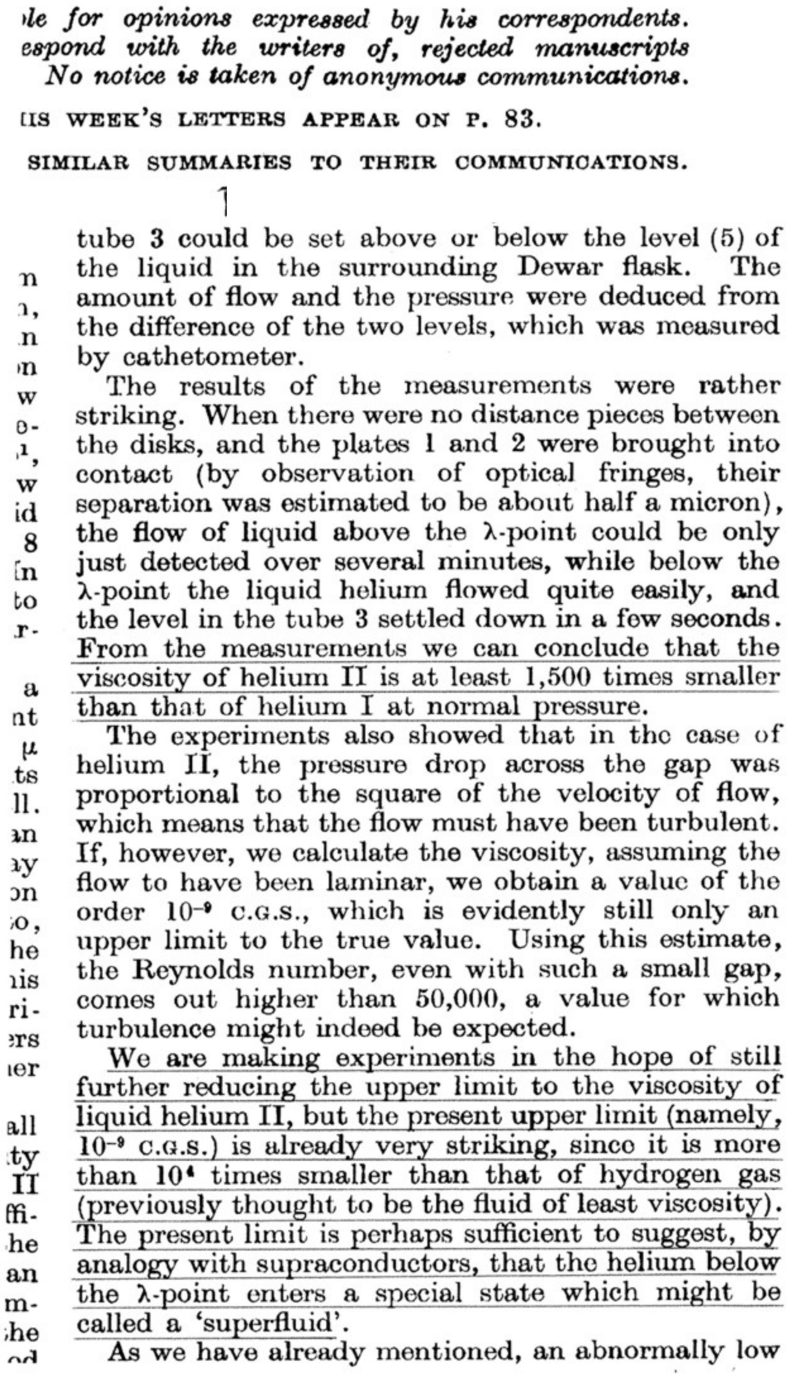
Institute for Physical Problems,
Academy of Sciences,
Moscow.

Dec. 3.

---

[1] Burton, NATURE, **135**, 265 (1935); Wilhelm, Misener and Clark, *Proc. Roy. Soc.*, A, **151**, 342 (1935).

[2] NATURE, **140**, 62 (1937).



le for opinions expressed by his correspondents.
espond with the writers of, rejected manuscripts
No notice is taken of anonymous communications.

IS WEEK'S LETTERS APPEAR ON P. 83.

SIMILAR SUMMARIES TO THEIR COMMUNICATIONS.

1

tube 3 could be set above or below the level (5) of the liquid in the surrounding Dewar flask. The amount of flow and the pressure were deduced from the difference of the two levels, which was measured by cathetometer.

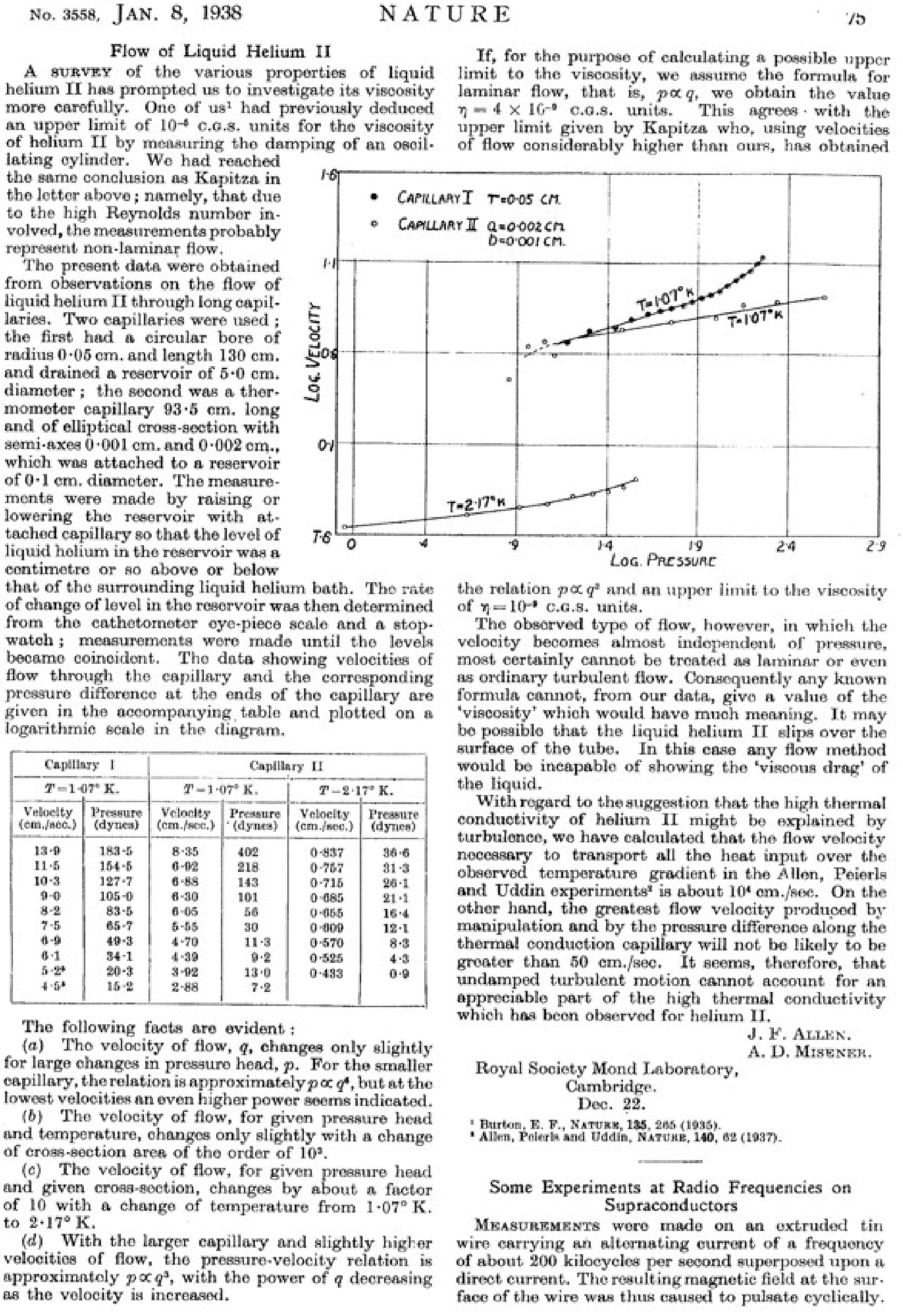
The results of the measurements were rather striking. When there were no distance pieces between the disks, and the plates 1 and 2 were brought into contact (by observation of optical fringes, their separation was estimated to be about half a micron), the flow of liquid above the λ-point could be only just detected over several minutes, while below the λ-point the liquid helium flowed quite easily, and the level in the tube 3 settled down in a few seconds. From the measurements we can conclude that the viscosity of helium II is at least 1,500 times smaller than that of helium I at normal pressure.

The experiments also showed that in the case of helium II, the pressure drop across the gap was proportional to the square of the velocity of flow, which means that the flow must have been turbulent. If, however, we calculate the viscosity, assuming the flow to have been laminar, we obtain a value of the order 10^{-9} C.G.S., which is evidently still only an upper limit to the true value. Using this estimate, the Reynolds number, even with such a small gap, comes out higher than 50,000, a value for which turbulence might indeed be expected.

We are making experiments in the hope of still further reducing the upper limit to the viscosity of liquid helium II, but the present upper limit (namely, 10^{-9} C.G.S.) is already very striking, since it is more than 10^4 times smaller than that of hydrogen gas (previously thought to be the fluid of least viscosity). The present limit is perhaps sufficient to suggest, by analogy with supraconductors, that the helium below the λ-point enters a special state which might be called a 'superfluid'.

As we have already mentioned, an abnormally low

Submitted
Dec 3, 1937
Published
Jan 8, 1938



Flow of Liquid Helium II

A SURVEY of the various properties of liquid helium II has prompted us to investigate its viscosity more carefully. One of us[1] had previously deduced an upper limit of 10^{-6} c.g.s. units for the viscosity of helium II by measuring the damping of an oscillating cylinder. We had reached the same conclusion as Kapitza in the letter above; namely, that due to the high Reynolds number involved, the measurements probably represent non-laminar flow.

The present data were obtained from observations on the flow of liquid helium II through long capillaries. Two capillaries were used; the first had a circular bore of radius 0·05 cm. and length 130 cm. and drained a reservoir of 5·0 cm. diameter; the second was a thermometer capillary 93·5 cm. long and of elliptical cross-section with semi-axes 0·001 cm. and 0·002 cm., which was attached to a reservoir of 0·1 cm. diameter. The measurements were made by raising or lowering the reservoir with attached capillary so that the level of liquid helium in the reservoir was a centimetre or so above or below that of the surrounding liquid helium bath. The rate of change of level in the reservoir was then determined from the cathetometer eye-piece scale and a stop-watch; measurements were made until the levels became coincident. The data showing velocities of flow through the capillary and the corresponding pressure difference at the ends of the capillary are given in the accompanying table and plotted on a logarithmic scale in the diagram.

Capillary I		Capillary II			
T = 1·07° K.		T = 1·07° K.		T = 2·17° K.	
Velocity (cm./sec.)	Pressure (dynes)	Velocity (cm./sec.)	Pressure (dynes)	Velocity (cm./sec.)	Pressure (dynes)
13·9	183·5	8·35	402	0·837	36·6
11·5	154·5	6·92	218	0·757	31·3
10·3	127·7	6·88	143	0·715	26·1
9·0	105·0	6·30	101	0·685	21·1
8·2	83·5	6·05	56	0·655	16·4
7·5	65·7	5·55	30	0·609	12·1
6·9	49·3	4·70	11·3	0·570	8·3
6·1	34·1	4·39	9·2	0·525	4·3
5·2*	20·3	3·92	13·0	0·433	0·9
4·5*	15·2	2·88	7·2		

The following facts are evident:

(*a*) The velocity of flow, q, changes only slightly for large changes in pressure head, p. For the smaller capillary, the relation is approximately $p \propto q^4$, but at the lowest velocities an even higher power seems indicated.

(*b*) The velocity of flow, for given pressure head and temperature, changes only slightly with a change of cross-section area of the order of 10^3.

(*c*) The velocity of flow, for given pressure head and given cross-section, changes by about a factor of 10 with a change of temperature from 1·07° K. to 2·17° K.

(*d*) With the larger capillary and slightly higher velocities of flow, the pressure-velocity relation is approximately $p \propto q^3$, with the power of q decreasing as the velocity is increased.

If, for the purpose of calculating a possible upper limit to the viscosity, we assume the formula for laminar flow, that is, $p \propto q$, we obtain the value $\eta = 4 \times 10^{-9}$ c.g.s. units. This agrees with the upper limit given by Kapitza who, using velocities of flow considerably higher than ours, has obtained the relation $p \propto q^2$ and an upper limit to the viscosity of $\eta = 10^{-9}$ c.g.s. units.

The observed type of flow, however, in which the velocity becomes almost independent of pressure, most certainly cannot be treated as laminar or even as ordinary turbulent flow. Consequently any known formula cannot, from our data, give a value of the 'viscosity' which would have much meaning. It may be possible that the liquid helium II slips over the surface of the tube. In this case any flow method would be incapable of showing the 'viscous drag' of the liquid.

With regard to the suggestion that the high thermal conductivity of helium II might be explained by turbulence, we have calculated that the flow velocity necessary to transport all the heat input over the observed temperature gradient in the Allen, Peierls and Uddin experiments[2] is about 10^4 cm./sec. On the other hand, the greatest flow velocity produced by manipulation and by the pressure difference along the thermal conduction capillary will not be likely to be greater than 50 cm./sec. It seems, therefore, that undamped turbulent motion cannot account for an appreciable part of the high thermal conductivity which has been observed for helium II.

J. F. ALLEN.
A. D. MISENER.

Royal Society Mond Laboratory,
Cambridge.
Dec. 22.

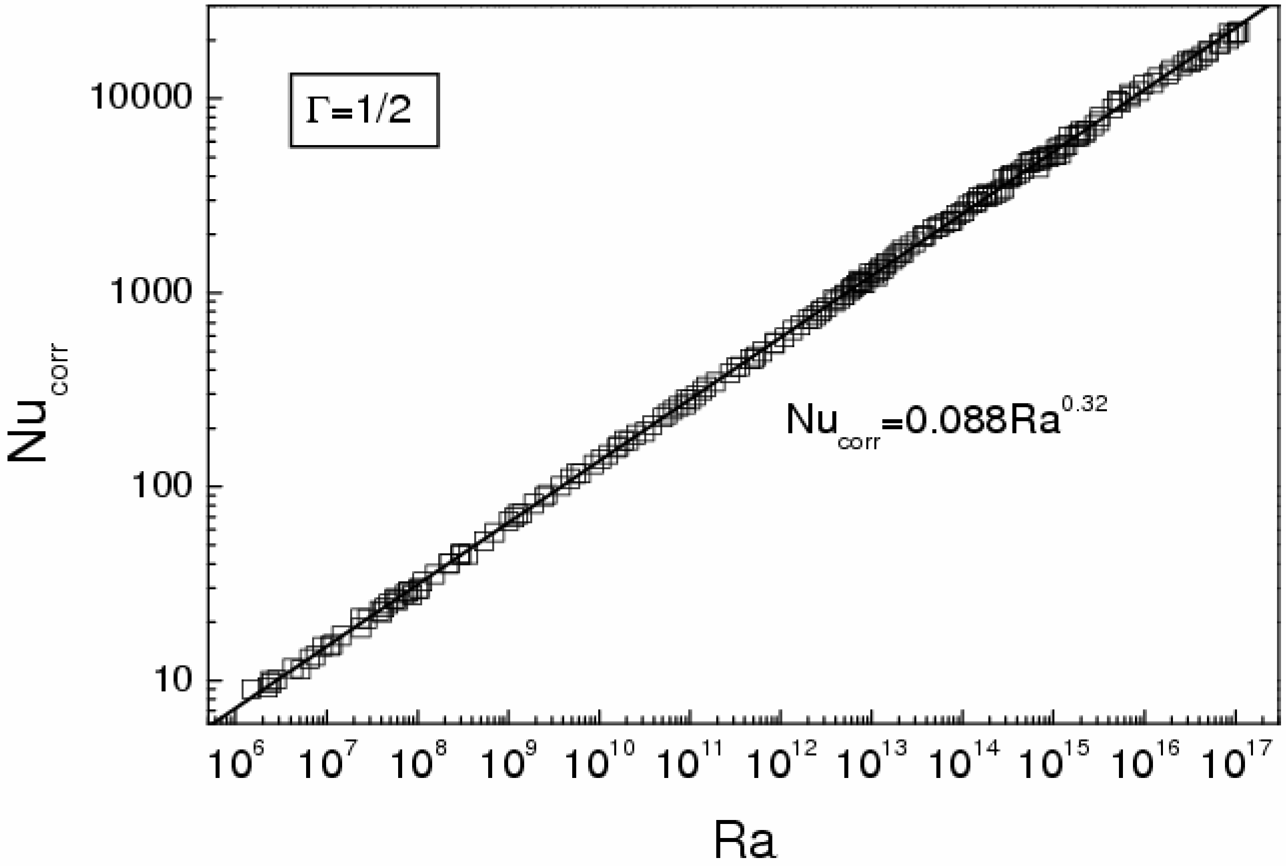
---

[1] Burton, E. F., NATURE, 135, 265 (1935).
[2] Allen, Peierls and Uddin, NATURE, 140, 62 (1937).

Some Experiments at Radio Frequencies on Supraconductors

MEASUREMENTS were made on an extruded tin wire carrying an alternating current of a frequency of about 200 kilocycles per second superposed upon a direct current. The resulting magnetic field at the surface of the wire was thus caused to pulsate cyclically.

Published
Jan 8, 1938
Submitted
Dec 22, 1937



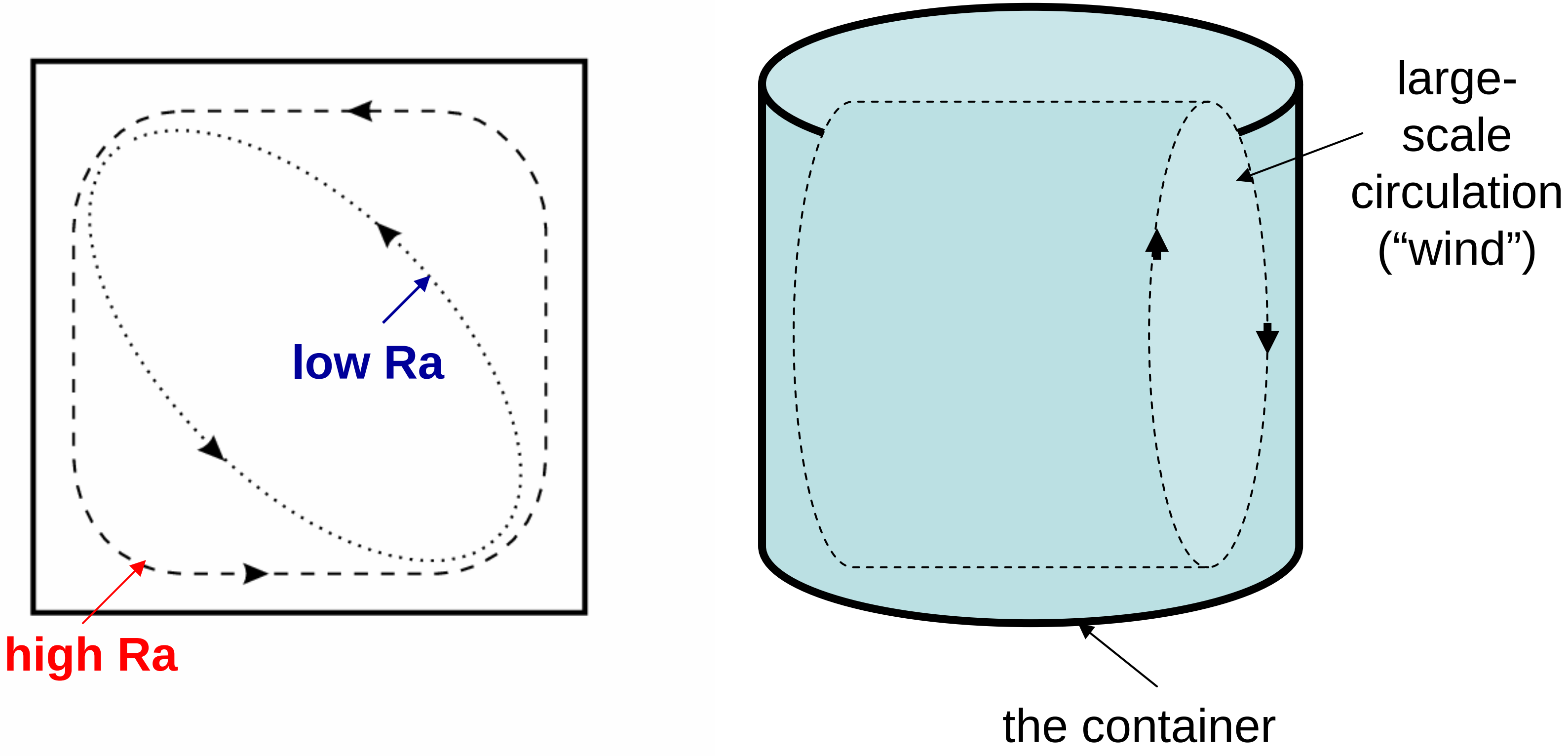
Niemela, Skrbek, Sreenivasan & Donnelly, *Nature* **404**, 837 (2000)
Slightly revised: Niemela & Sreenivasan, *J. Low Temp. Phys.* (2006)
[Pioneers: Threlfall (Cambridge); Libchaber, Kadanoff and coworkers (Chicago)]

selected summary of the theory for Nusselt number variation

$$Nu = CRa^{1/3}(\lambda/H)^{\beta}$$

λ is the thermal boundary layer thickness, O(H/[2Nu]). The exponent β cannot be determined by dimensional arguments.

Kraichnan (1962):	$\beta = -1/3$
Castaing et al. (1989):	$\beta = 1/6$
Grossmann & Lohse (2000):	no single power law
Traditional (1950's):	$\beta = 0$
(Priestley, Townsend, Malkus, Howard, …)	



the wind breaks the symmetry, with important consequences

plumes…

… and their self-organization into a large scale flow in a confined apparatus

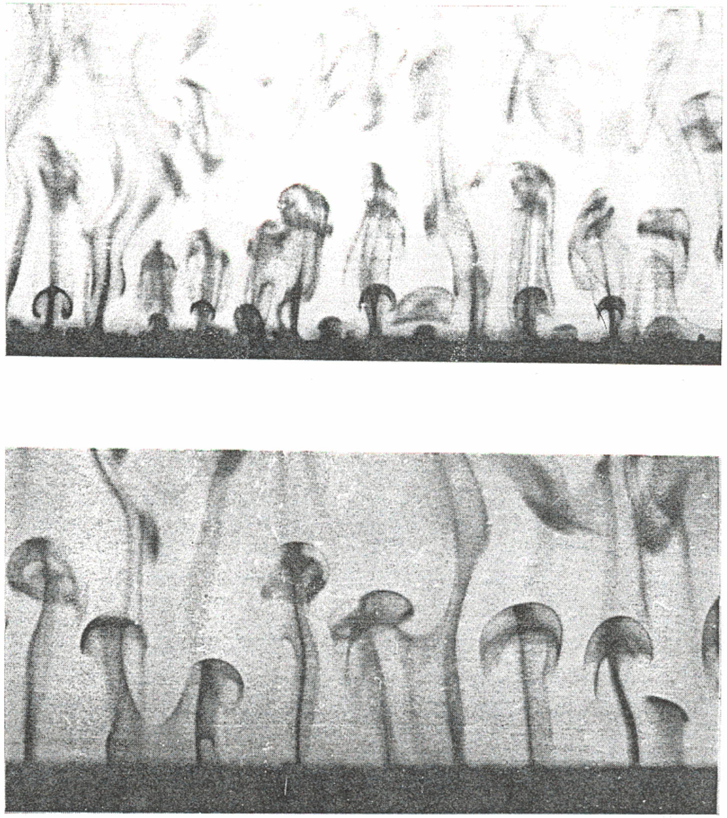
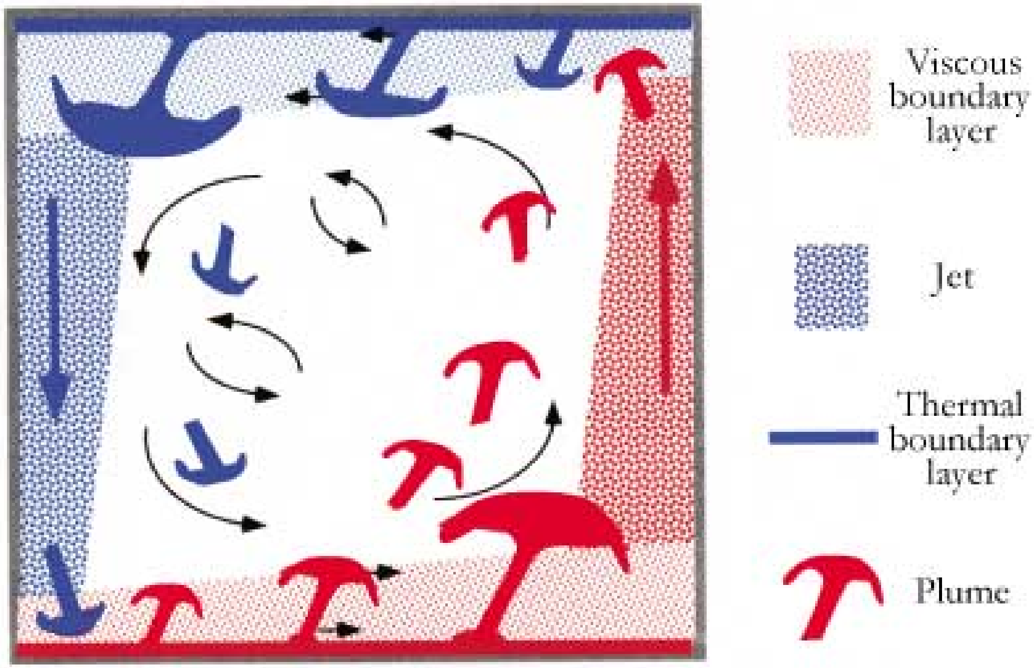


FIGURE 1. Photographs of thermals rising from a heated horizontal surface.

Sparrow, Husar & Goldstein

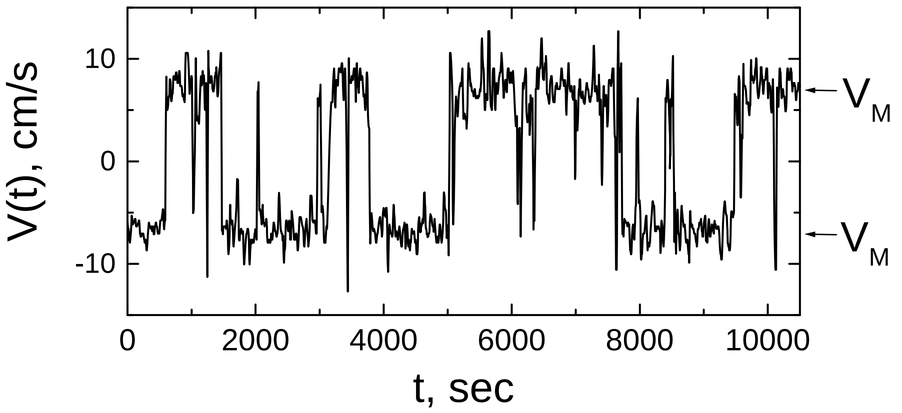
J. Fluid Mech. **41,** 793 (1970)



L. Kadanoff, *Phys. Today*, August 2001
(for flow visualization and quantitative work, see K.-Q. Xia et al. from Hong Kong)

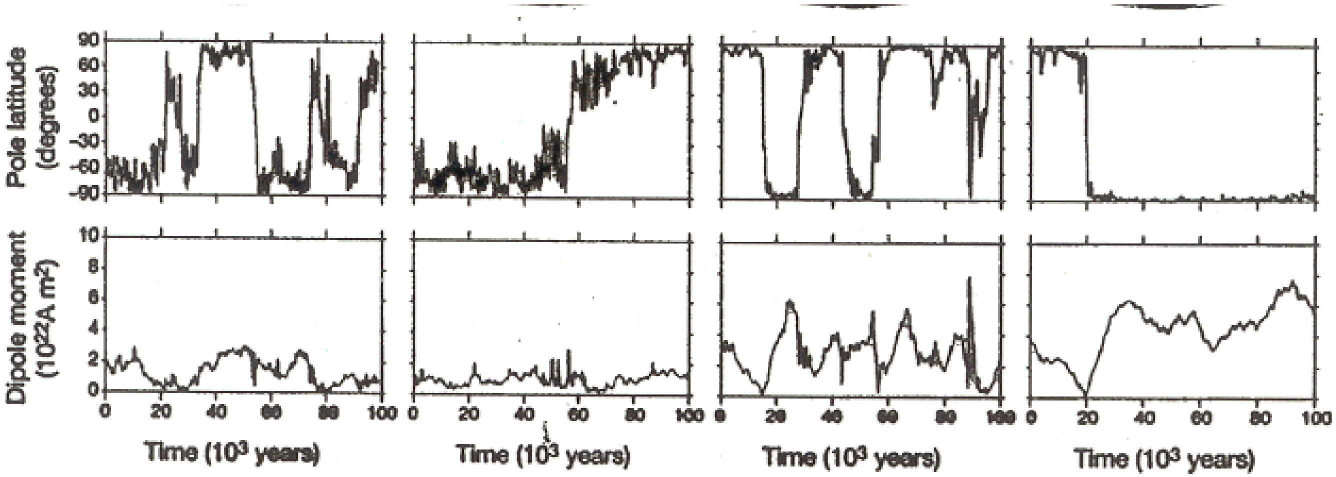
the large scale wind with occasional reversals

(Sreenivasan, Bershadskii & Niemela, PRE 65, 056306, 2002)



segment of continuous 120-hour record; $\Gamma = 1$

geomagnetic polarity reversals



Glatzmaier, Coe, Hongre & Roberts, *Nature* 401, 885-890 (1999)

How are the reversals distributed?

$\tau_1 =$ time between subsequent switches in the velocity signal

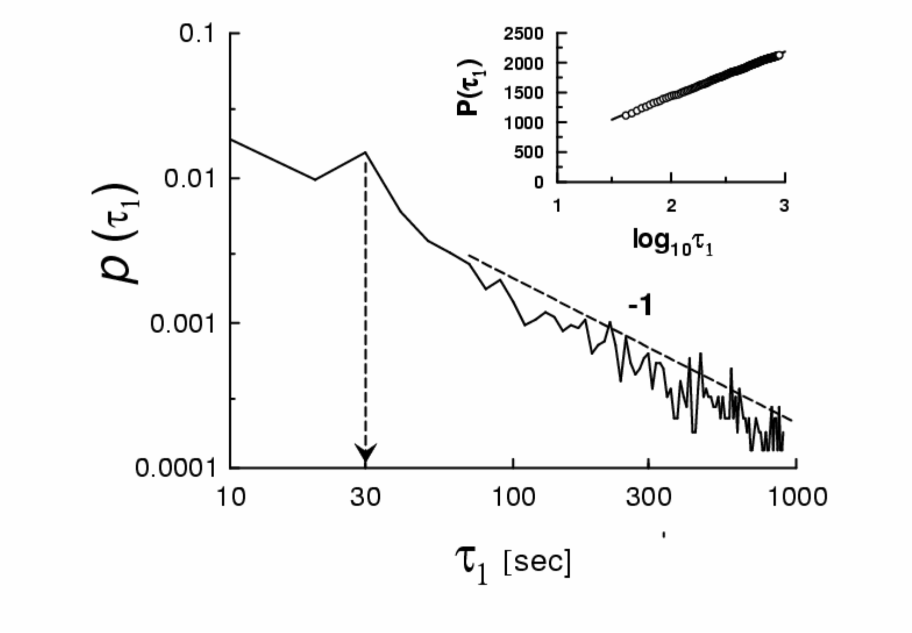
$$\tau_1 \equiv T_{n+1} - T_n$$

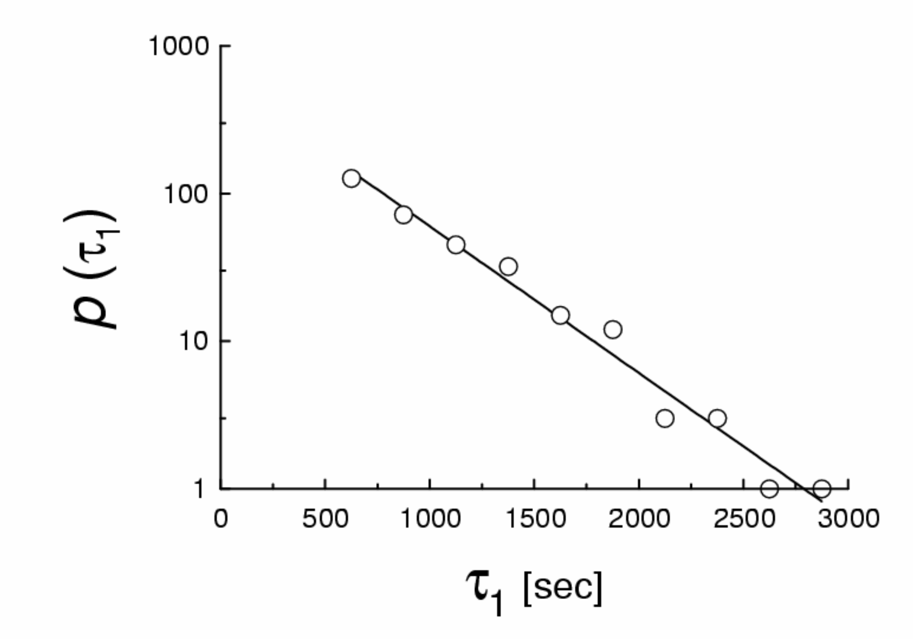
power-law scaling of the probability

density function for small τ_1

for large τ_1: $p(\tau_1) \sim \exp[-(\tau_1/\tau_m)]$

$\tau_m \approx 400s$

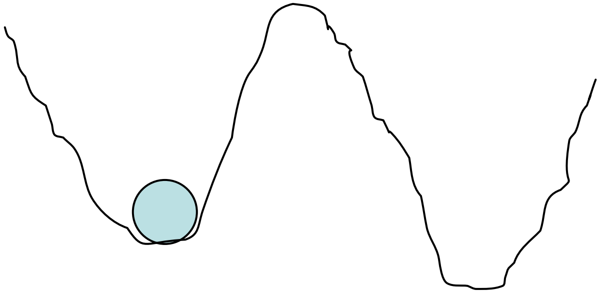




Sreenivasan, Bershadskii & Niemela, *Phys. Rev. E* **65**, 056306 (2002)

-1 power law scaling characteristic of SOC systems
(see papers in *Europhys. Lett., Physica A and PRE*)

Dynamical model

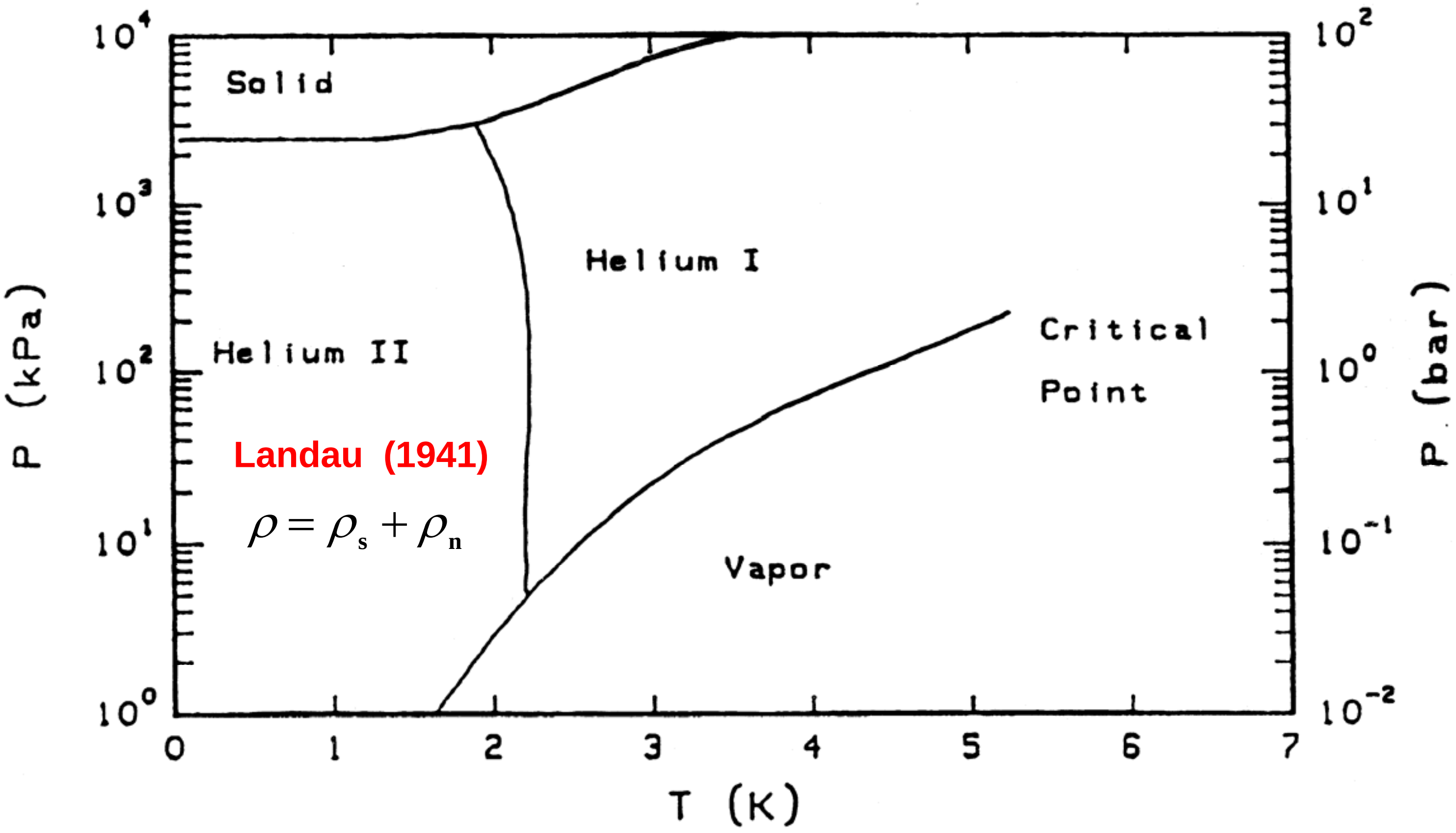


double-well potential

$$p(\tau_1) : \quad \exp[-(\tau_1 / \tau_m)]$$

Balance between buoyancy and friction, forced by stochastic noise

For certain combinations of parameters, one obtains power-law for small times and exponential distribution for large times.



Landau (1941)

$$\rho = \rho_s + \rho_n$$

Superfluid is irrotational. Under certain conditions, thin vortex filaments are spontaneously formed. The circulation is these filaments is quantized i.e., k = h/m (Onsager 1949). The filaments are line “singularities” of about 1 angstrom in diameter (Feynman 1955).

… liquid helium…possesses a number of peculiar properties, the most important of which is superfluidity discovered by P.L. Kapitza….Tisza’s well-known [theoretical] attempt … cannot be accepted as satisfactory…

Landau (1941)

Thus, the well-known invariant called hydrodynamic circulation is quantized; the quantum of circulation is h/m.

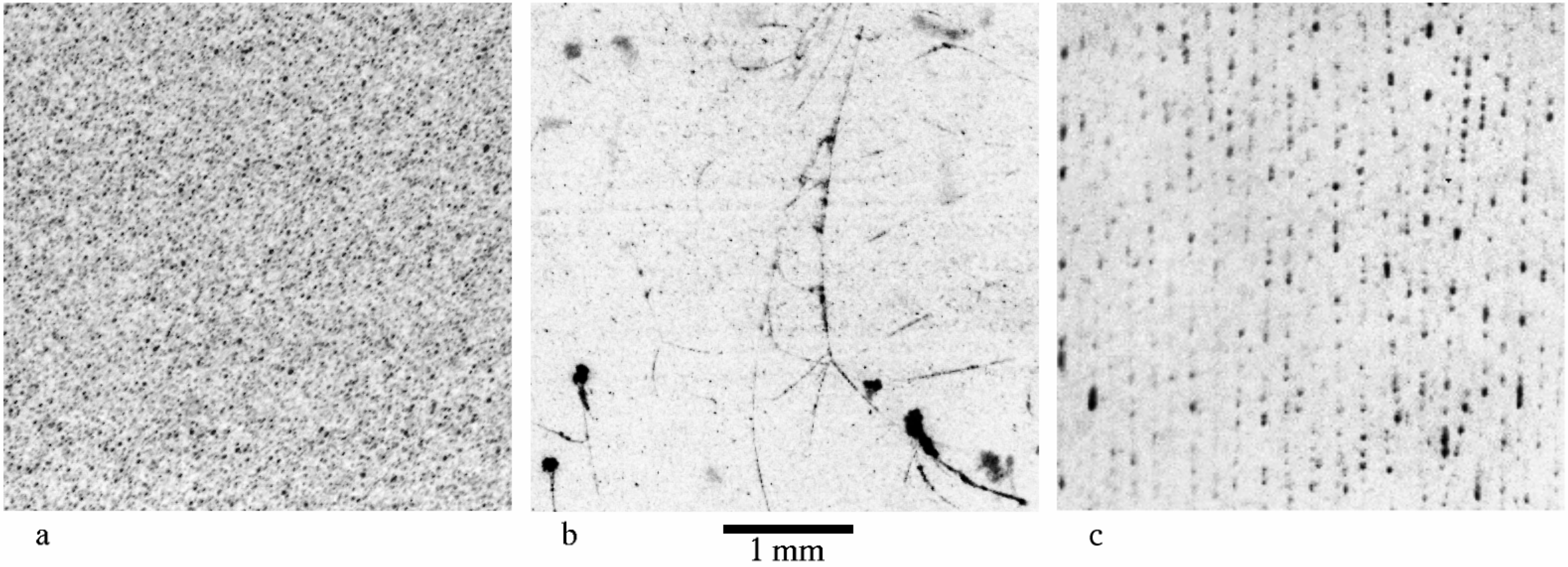
Onsager (1949)

Except for a few angstroms from the center of the core, the laws obeyed are those of classical hydrodynamics.

Feynman (1955)

never seen until now experimentally

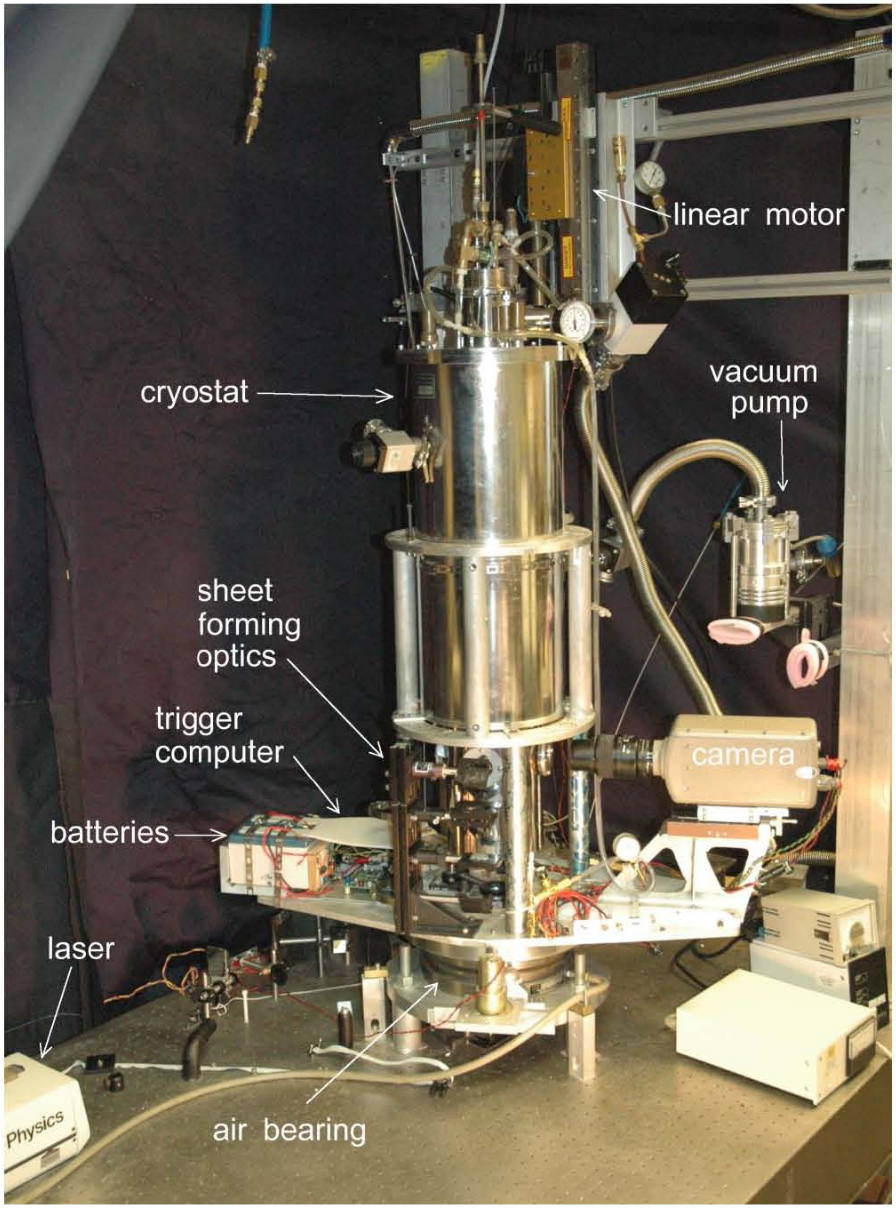
50 years later…

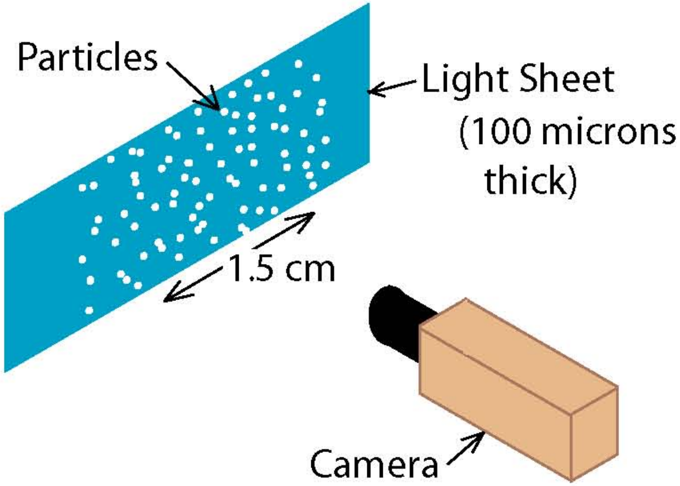


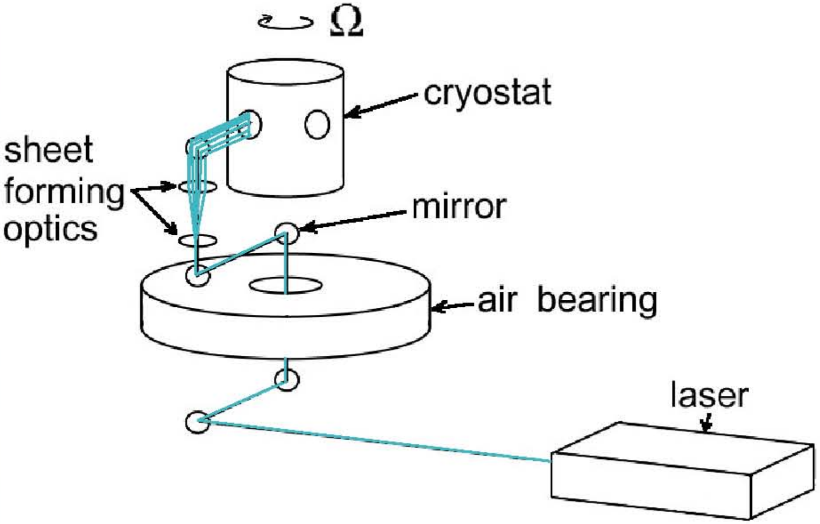
Suspension of hydrogen particles (a) just above the lambda temperature, and (b) just below, where some particles have collected along branching filaments. In (c), particles arrange themselves along vertical lines when the system rotates steadily about the vertical axis. The spacing of lines is remarkably uniform.

G.P. Bewley, D.P. Lathrop & K.R. Sreenivasan, *Nature* 441, 558 (2006); also Experiments in Fluids (submitted, 2006)

Apparatus

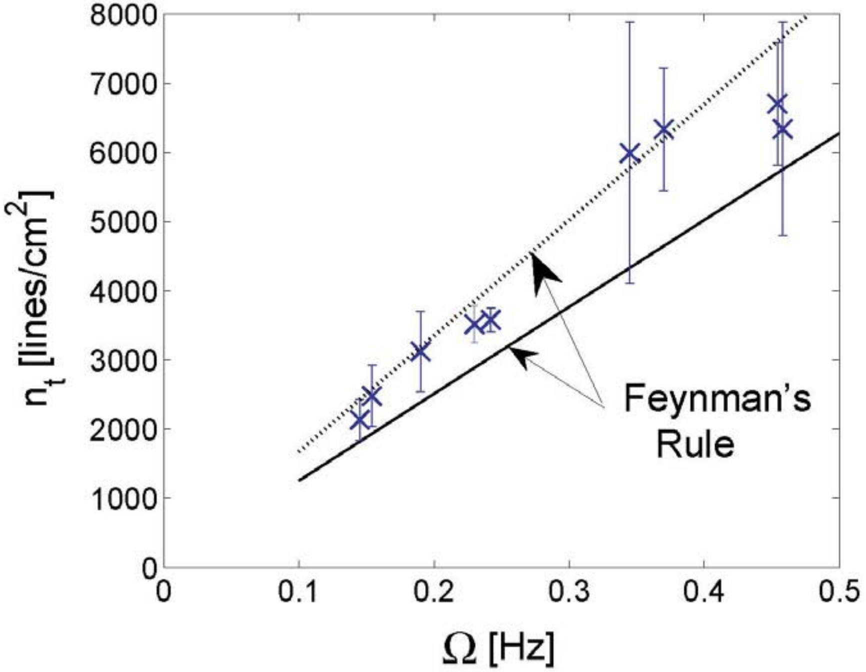




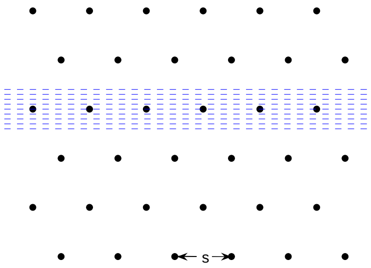


Laser Beam routing

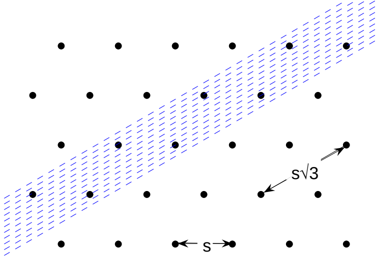
Lattice density

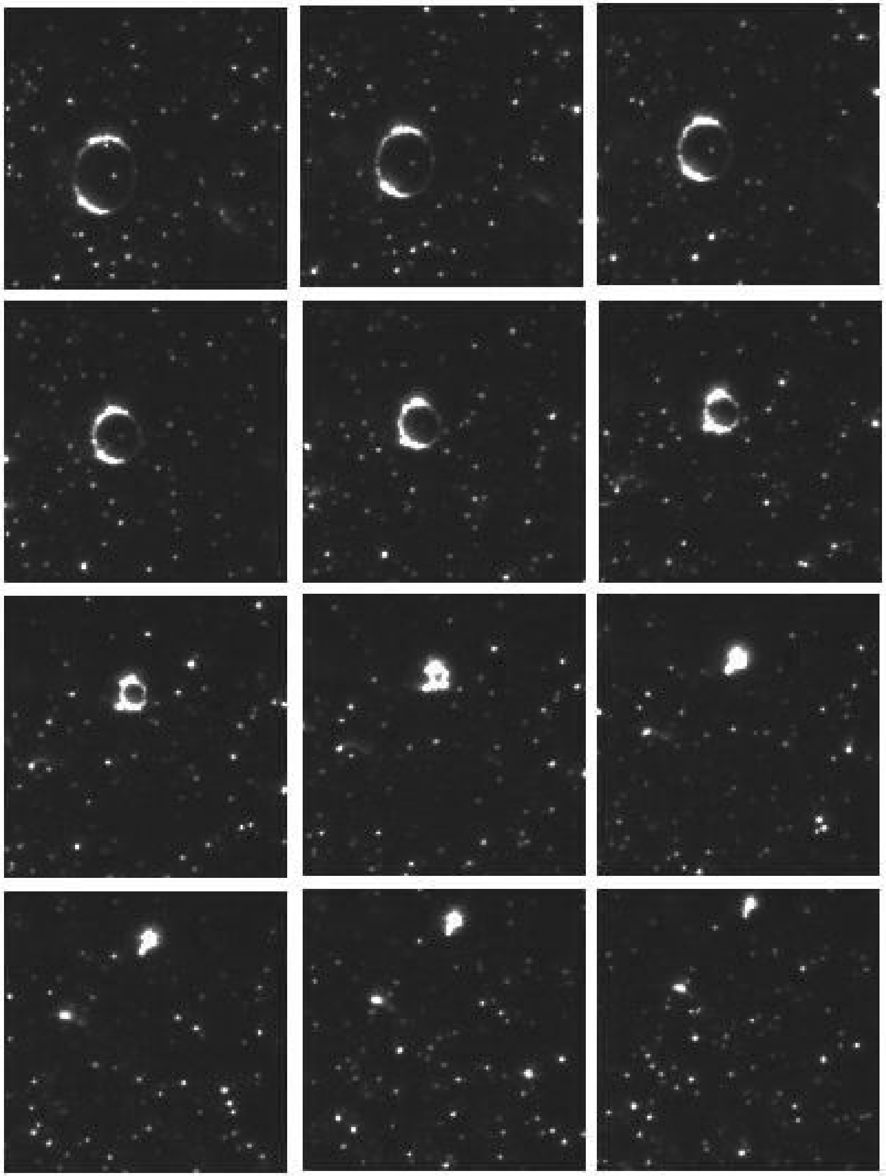


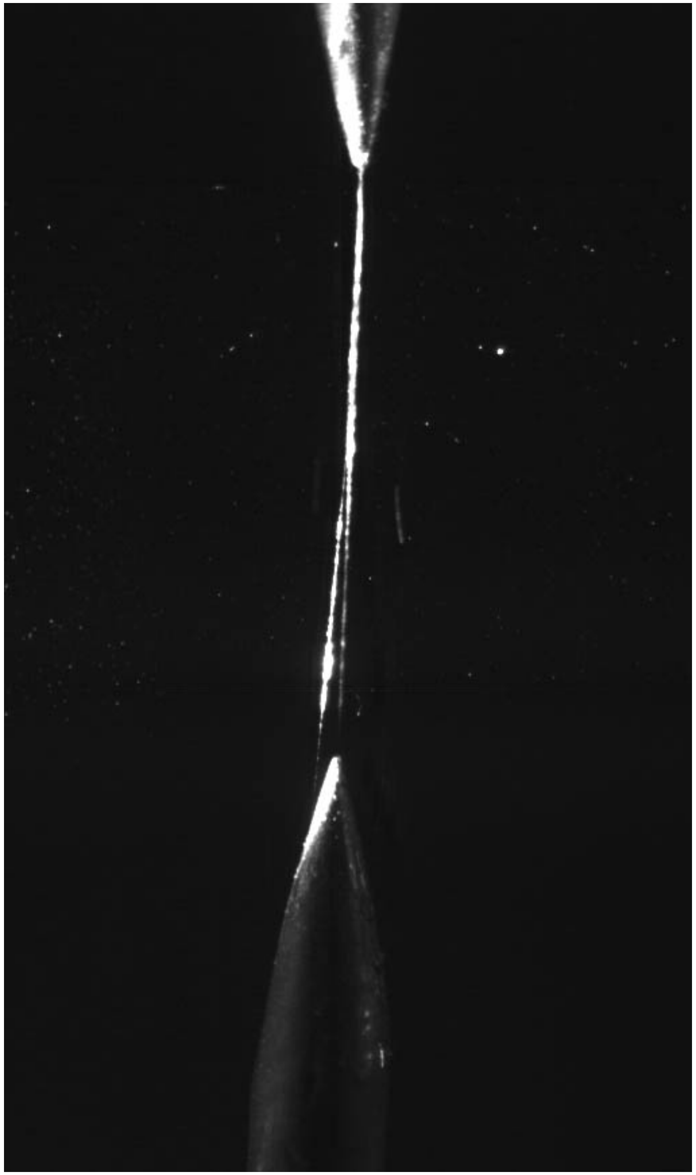
R.P. Feynman (1955)
***Prog. Low Temp. Phys.* 1,** 17

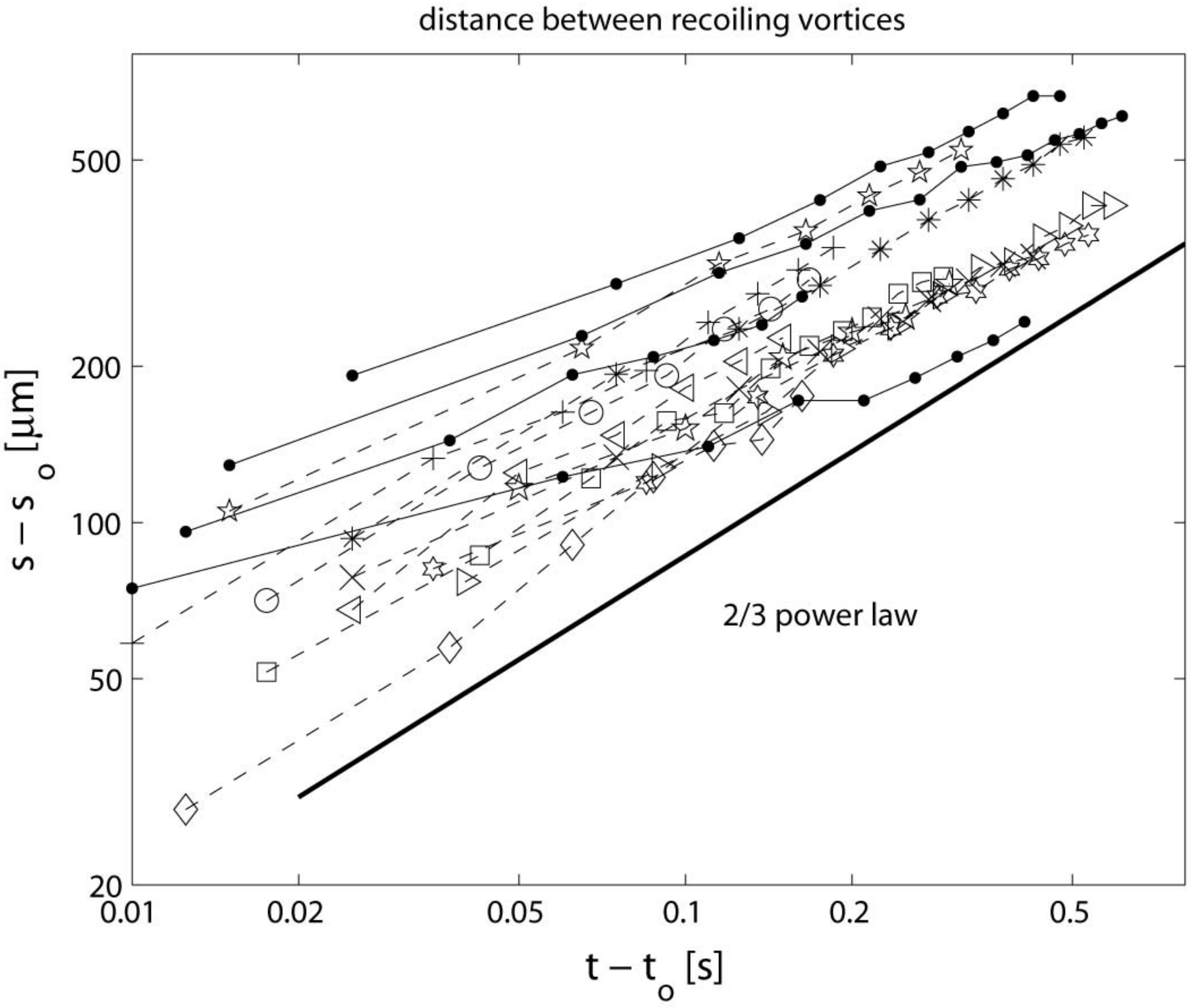


$n \approx 2000\Omega$ lines/cm^2








distance between recoiling vortices
s − s_o [μm]
t − t_o [s]
2/3 power law
20
50
100
200
500
0.01
0.02
0.05
0.1
0.2
0.5

Previous observation of quantized vortices

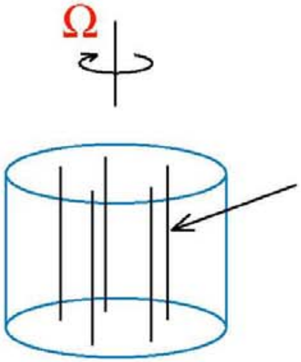
Yarmchuk, E.J., Gordon, M.J.V. and Packard, R.E. (1979),
Phys. Rev. Lett. **43**, 214-217.

Rotating superfluid:

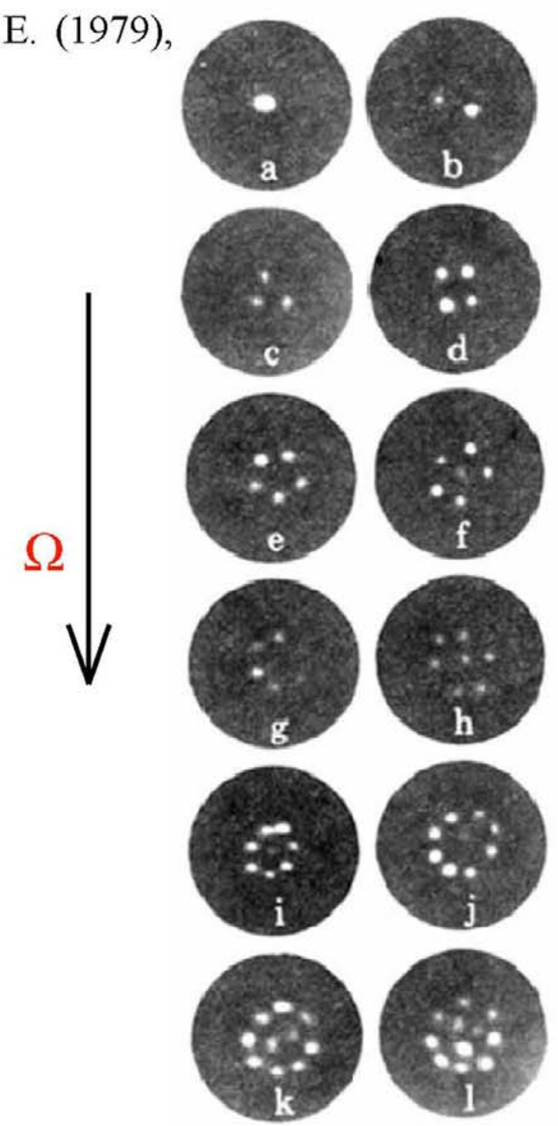
$n_o \approx 2000\Omega$ lines/cm^2

Feynman's rule

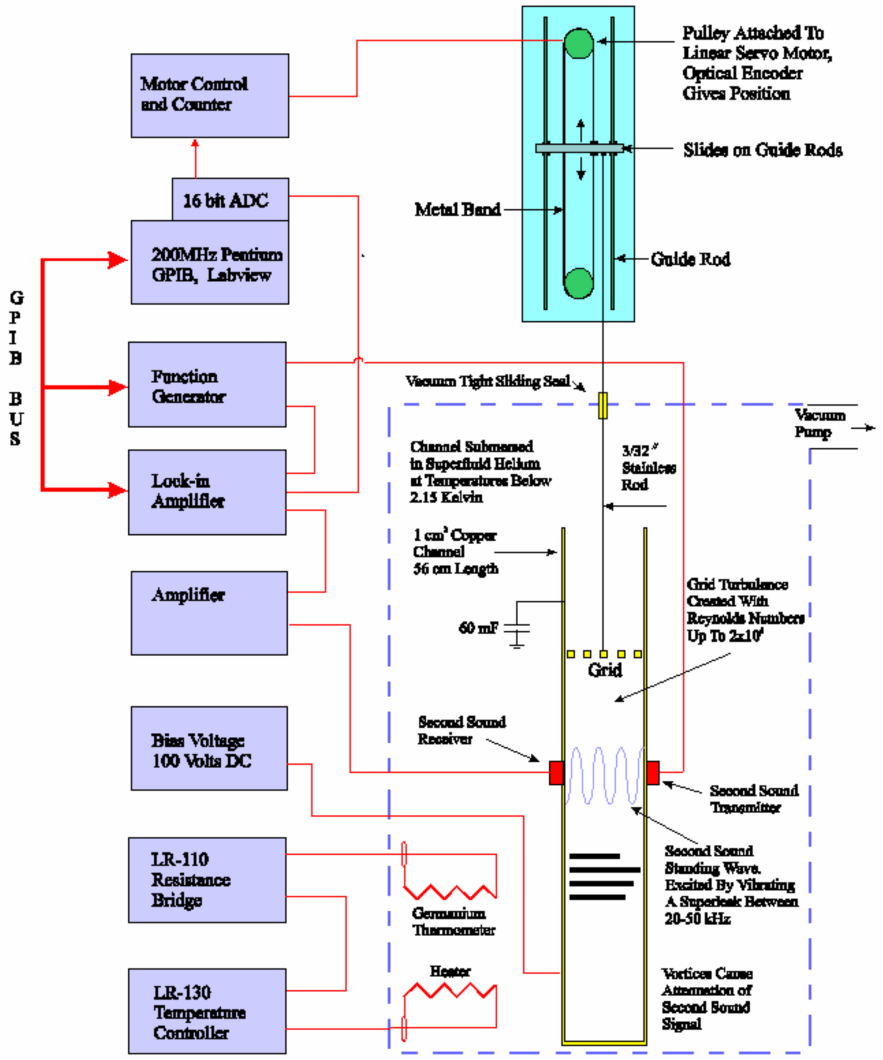
R.P. Feynman, *Prog. Low Temp. Phys.* (1955)



Quantized vortex



not suitable for visualizing random vortices; simulations by Schwarz, Tsubota

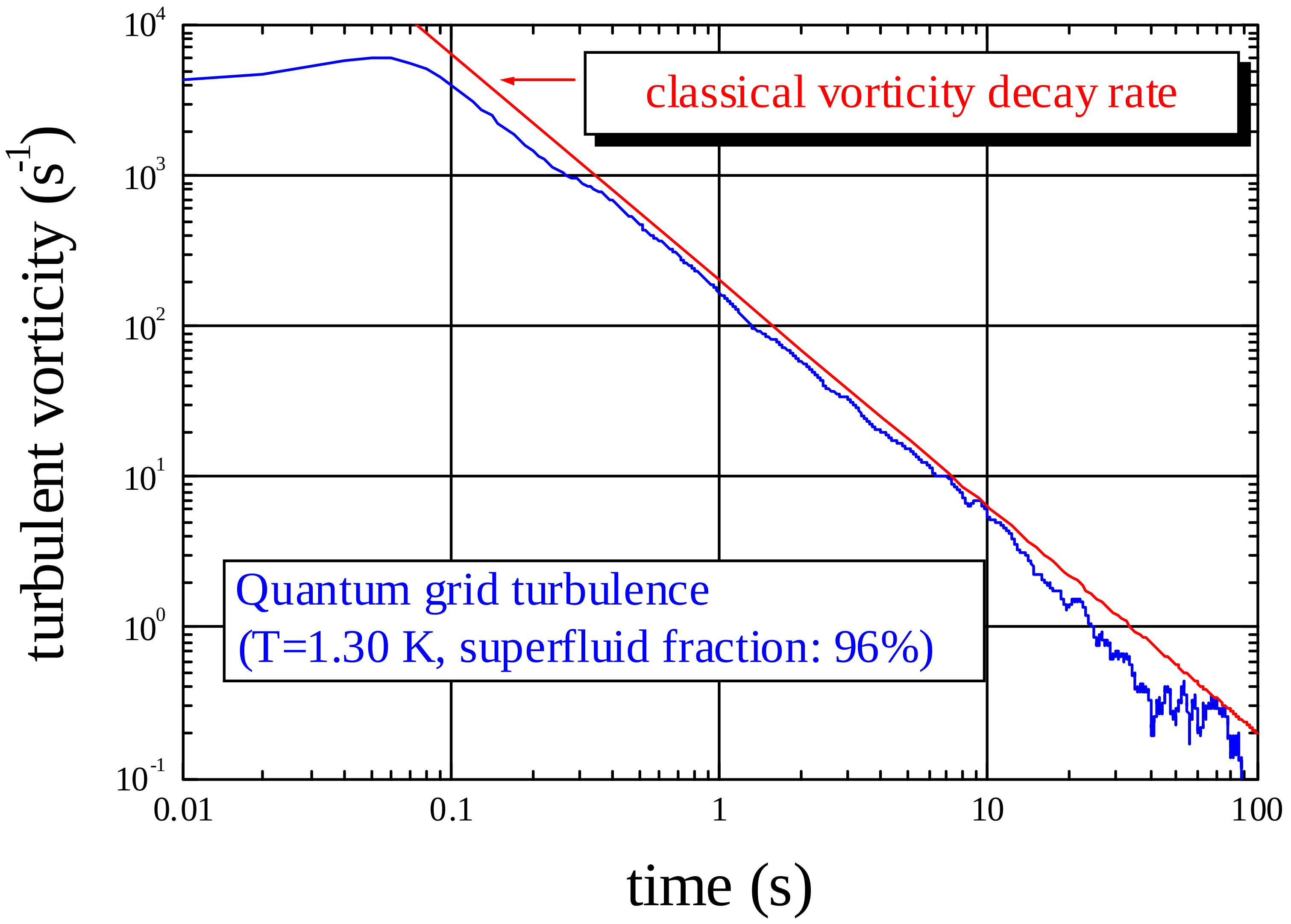


1 cm X 1 cm

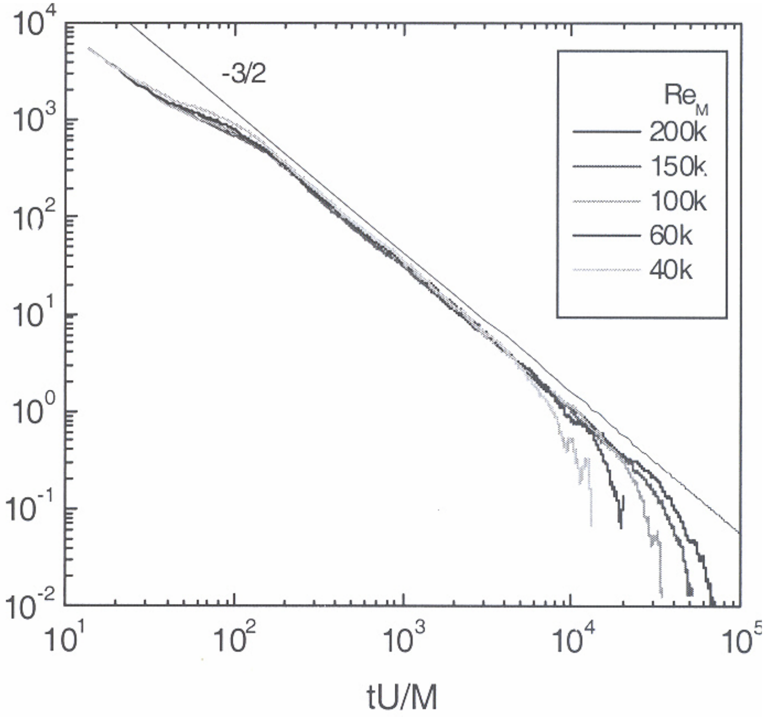
$R_M = 40$ K to 200 K

Niemela & Sreenivasan,
J. Low Temp. Phys. 2007

the apparatus for helium II grid turbulence
(Donnelly, Stalp, Niemela, Skrbek, KRS, etc)



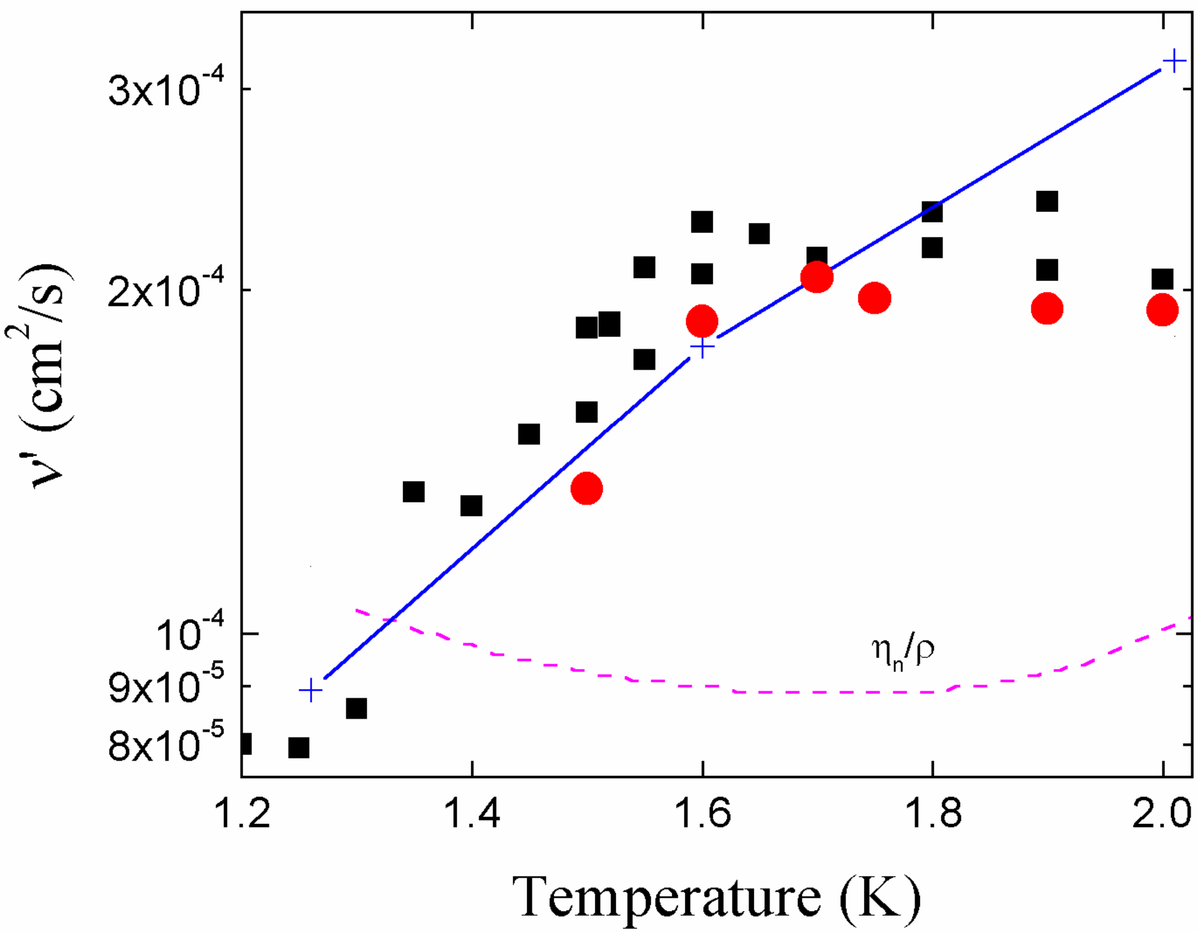
Superfluid vorticity: $L \propto \langle \omega^2 \rangle_s^{1/2}$



Because the superfluid vorticity decays as $t^{-3/2}$, just as does classical vorticity, and the observed prefactors are as expected, <u>the notion arises that the two turbulence fields are coupled</u> (except at dissipative scales where the mechanisms are different).

"Hypothesis of coupled vorticity" (Sreenivasan & Donnelly, *Adv. Appl. Mech.* 2002; Goldenfeld, Berenghi, Vinen, Volovich, etc)

Superfluid turbulence occurs when the classical turbulence occurs.



effective viscosity as a function of temperature: Vinen & Niemela (2005)

V.L. Ginzburg (2003 Nobel Prize in Physics)

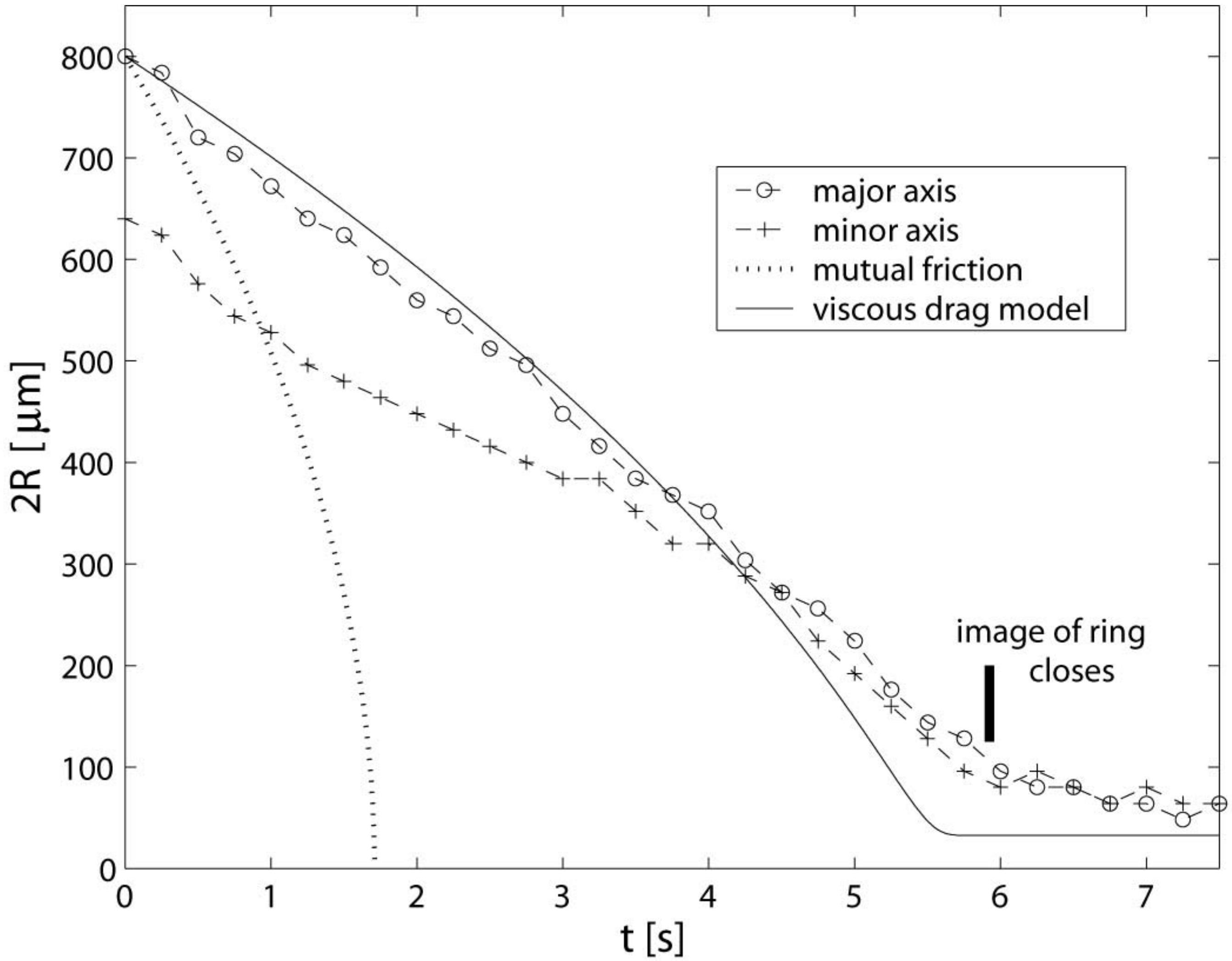
(*Physics Today*, May 1990, page 9; also *Uspekhi* **42** (4), 353, 1999)

- Classified Physics into Microphysics, Astrophysics and Macrophysics (the small, the large and the complex)
- One of the 11 items of Macrophysics: “Strongly Nonlinear Phenomena: Turbulence”

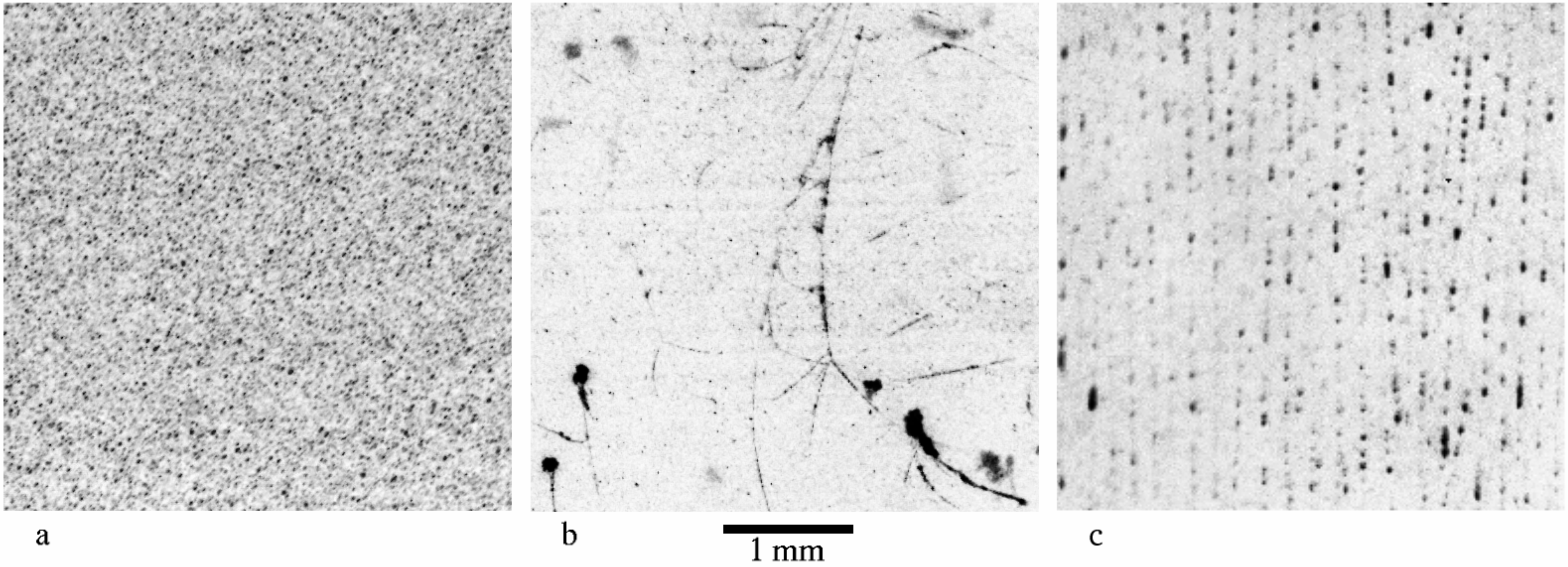
Concluding Remarks

- To understand mesoscale physics, or the physics of complexity, controlled experiments, accurate simulations and rigorous theory are all needed. Our intuition is not yet sufficiently well honed to come up with general laws.
- Turbulence offers an excellent paradigm of mesoscale physics.
- Using helium, the range of fluid-dynamic parameters has been extended as never before.
- Tremendous progress on convection has been possible, and some new physics indeed has been discovered. This part of the promise of helium has been realized. The new physics reveals that superfluidity cannot be used to further the Re-range because quantized vortices bring the effective kinematic viscosity to the levels of helium I.
- The resolution of small-scale turbulence has not been possible. This is a challenge for instrumentation. Existing tools such as hot wires, PIV, LDV, molecular (fluorescence) tracers, pressure sensors, ion trapping, have been upgraded (this is an entirely nontrivial task) but there is increasing awareness that other methods will be needed.
- The work offers great challenges for simulations.

The end


major axis
minor axis
mutual friction
viscous drag model
image of ring
closes
2R [µm]
t [s]
0
100
200
300
400
500
600
700
800
0
1
2
3
4
5
6
7

50 years later…



Suspension of hydrogen particles (a) just above the lambda temperature, and (b) just below, where some particles have collected along branching filaments. In (c), particles arrange themselves along vertical lines when the system rotates steadily about the vertical axis. The spacing of lines is remarkably uniform.

G.P. Bewley, D.P. Lathrop & K.R. Sreenivasan, *Nature* 441, 558 (2006); also Experiments in Fluids (submitted, 2006)

The energy decay of classical turbulence in homogeneous turbulence

$$\frac{d}{dt}\langle u^2\rangle = -\langle\varepsilon\rangle = \underset{\substack{\Uparrow \\ O(1)}}{C}\langle u^2\rangle^{3/2}/\Lambda$$

Λ grows with time as a power law and saturates: $\Lambda \propto d$.

Solve for $\langle u^2\rangle$, $\langle\varepsilon\rangle$ and $\langle\omega^2\rangle$, and get

$$\langle\omega^2\rangle^{1/2} \propto (d/\nu^{3/2})t^{-3/2}$$

From Kaneda et al., *Phys. Fluids* **15**, L21 (2003)

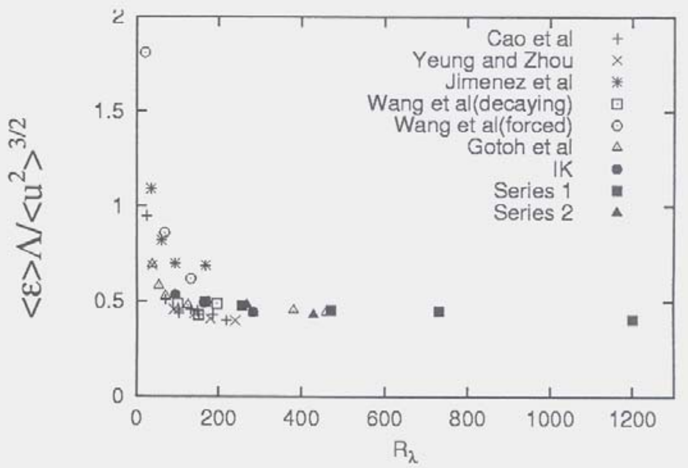


FIG. 3. Normalized energy dissipation rate D versus R_λ from Ref. 5 (data up to $R_\lambda = 250$), Ref. 3 (△,●), and the present DNS databases (■,▲).

Summary of motivation in broad terms

1. Helium at cryogenic temperatures has the smallest kinematic viscosity of all known fluids (typically, 10^{-3} times smaller than that of air at NTP).
2. Superfluidity of helium II (Kapitza 1938) suggests that new physics is likely.

He II = normal fluid + superfluid

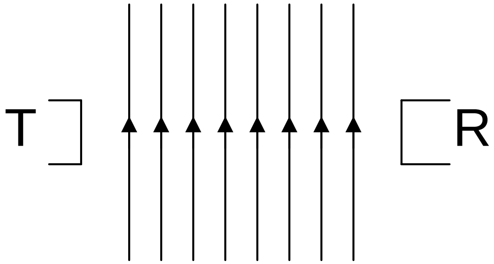
(NS = 0) (EE = 0)

The superfluid is irrotational except for quantized vortex filaments that are formed spontaneously. How?

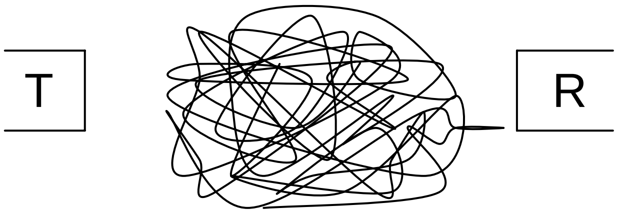
Their motion in the normal fluid is subject to laws of fluid mechanics. In particular, they suffer drag or “mutual friction” (Vinen & Hall 1956)

Under controlled conditions, quantized vortices are simple in geometry.

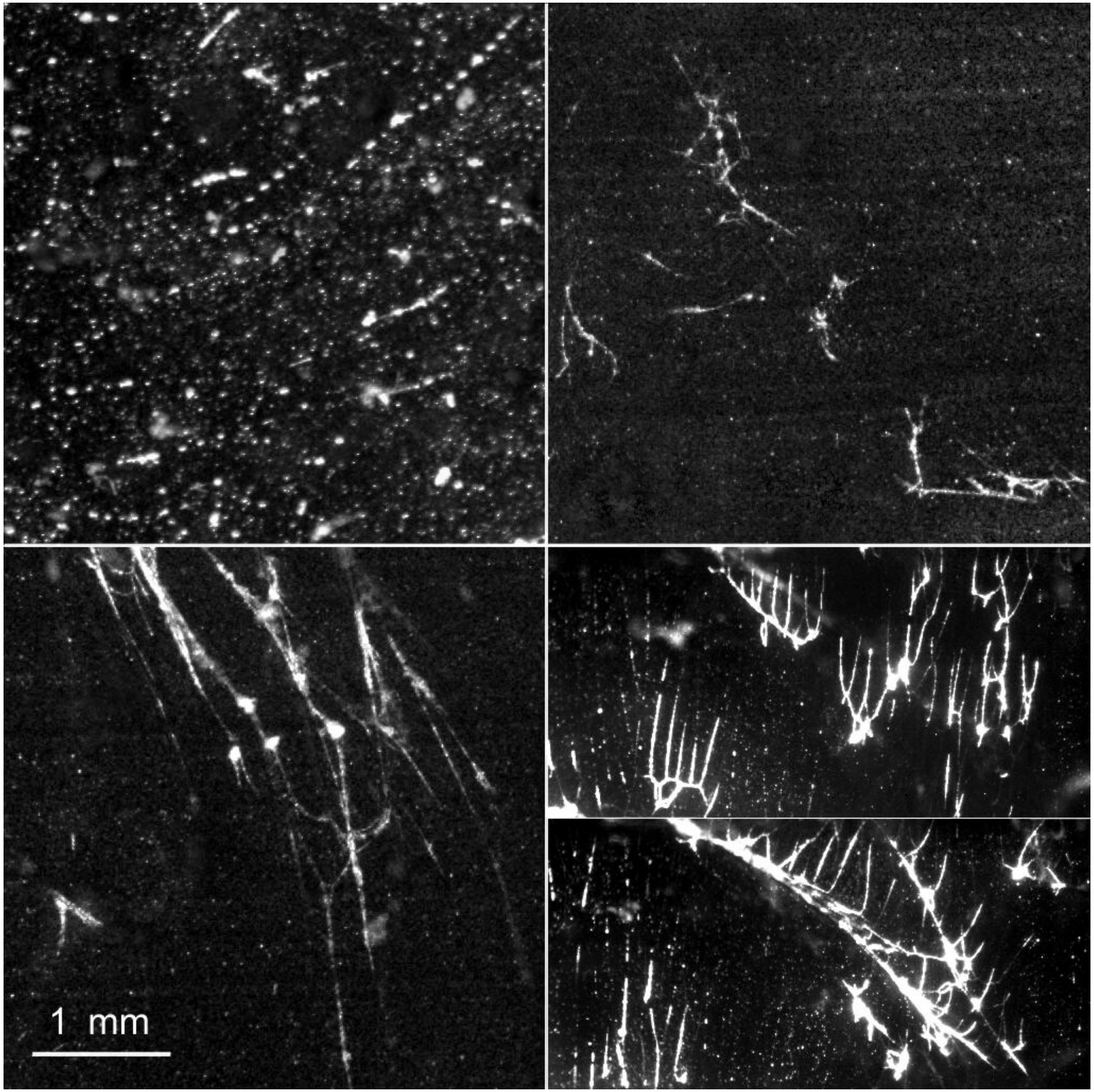
Attenuation of the second sound
is proportional to the “line length”, L



However, the vortices become a “tangle”
under less controlled conditions. This is
“superfluid turbulence”.



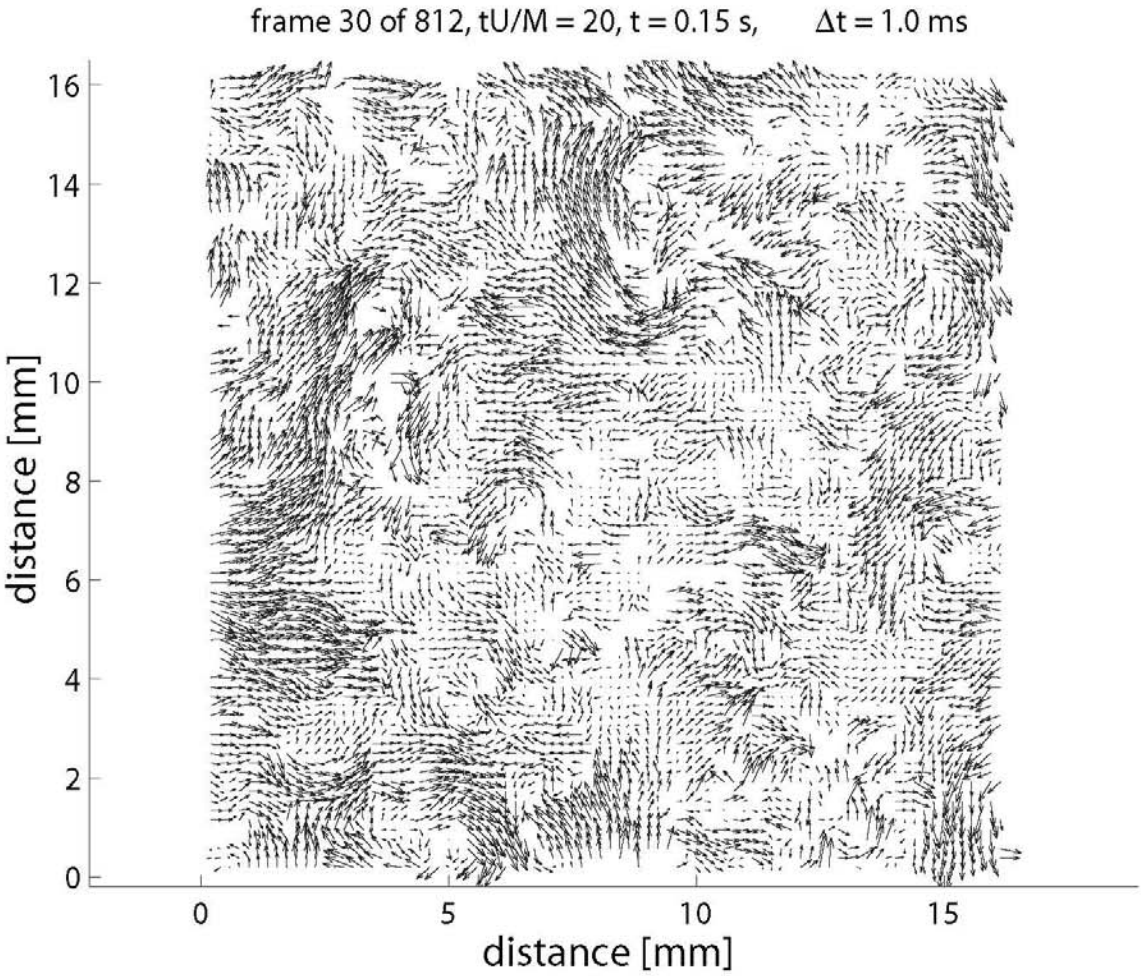
Attenuation depends on the projected
line length (proportional to rms vorticity)



different forms of superfluid vortices

What happens if we pull a grid through helium?

standard grid turbulence above T_λ

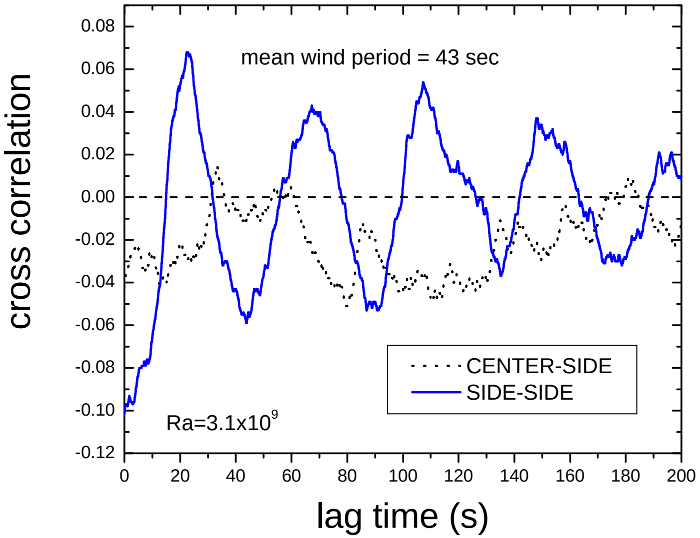


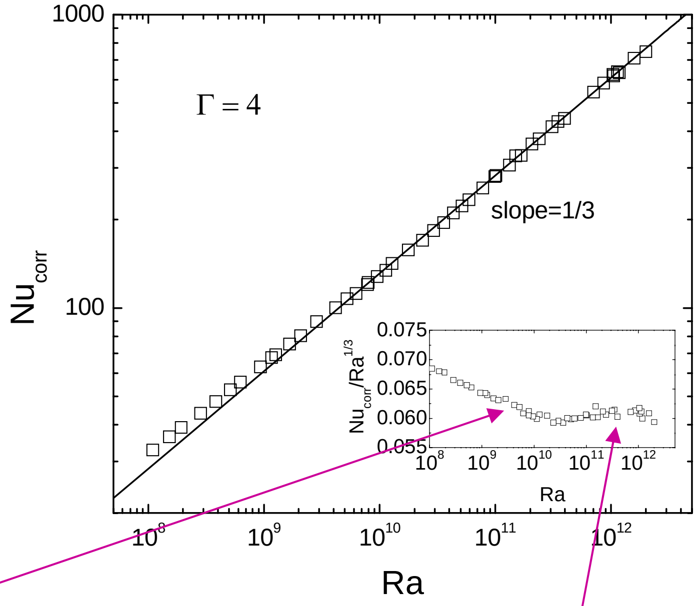
White, Karpetis & Sreenivasan, *J. Fluid Mech.* (2003)

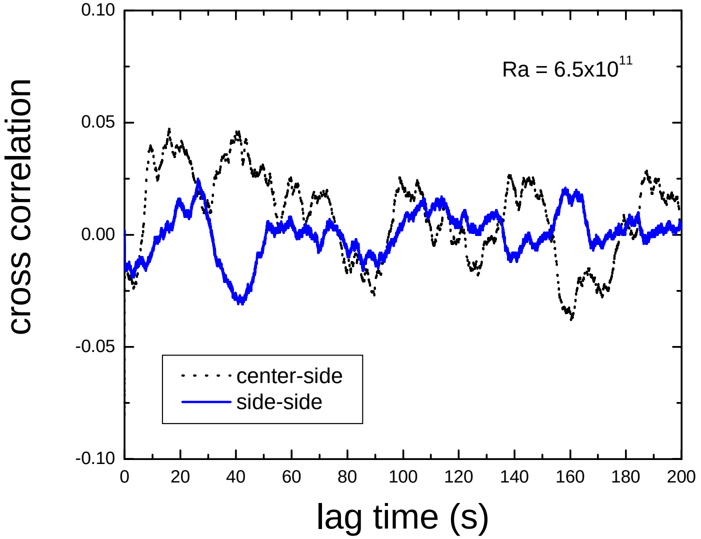
What about higher aspect ratio?

Nu under strict Boussinesq conditions

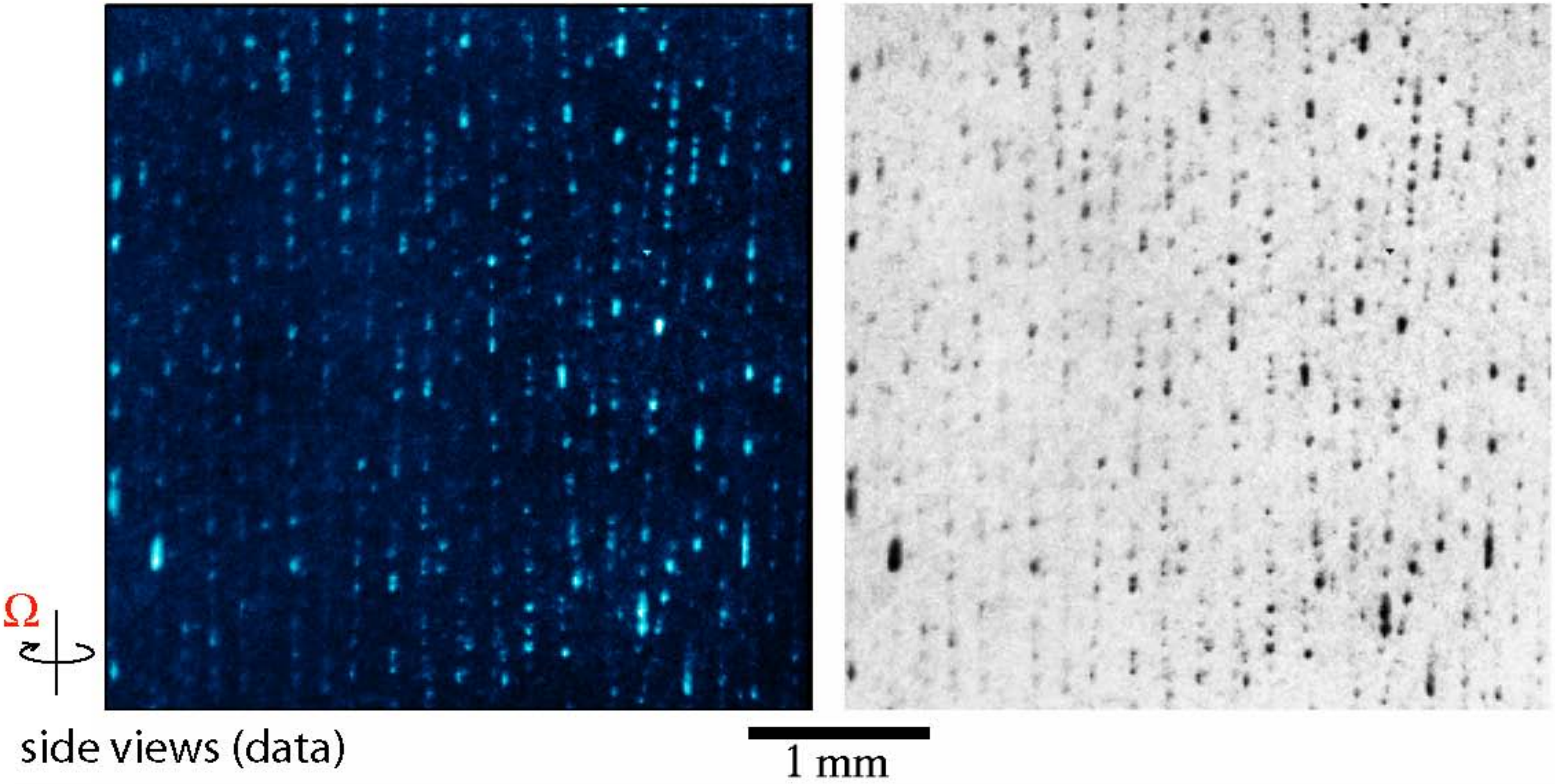
correlation of signals from opposite sides and that between the side and center





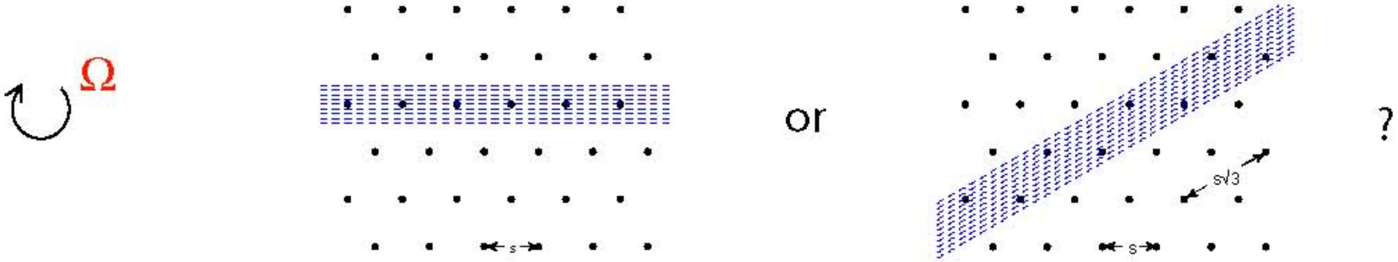


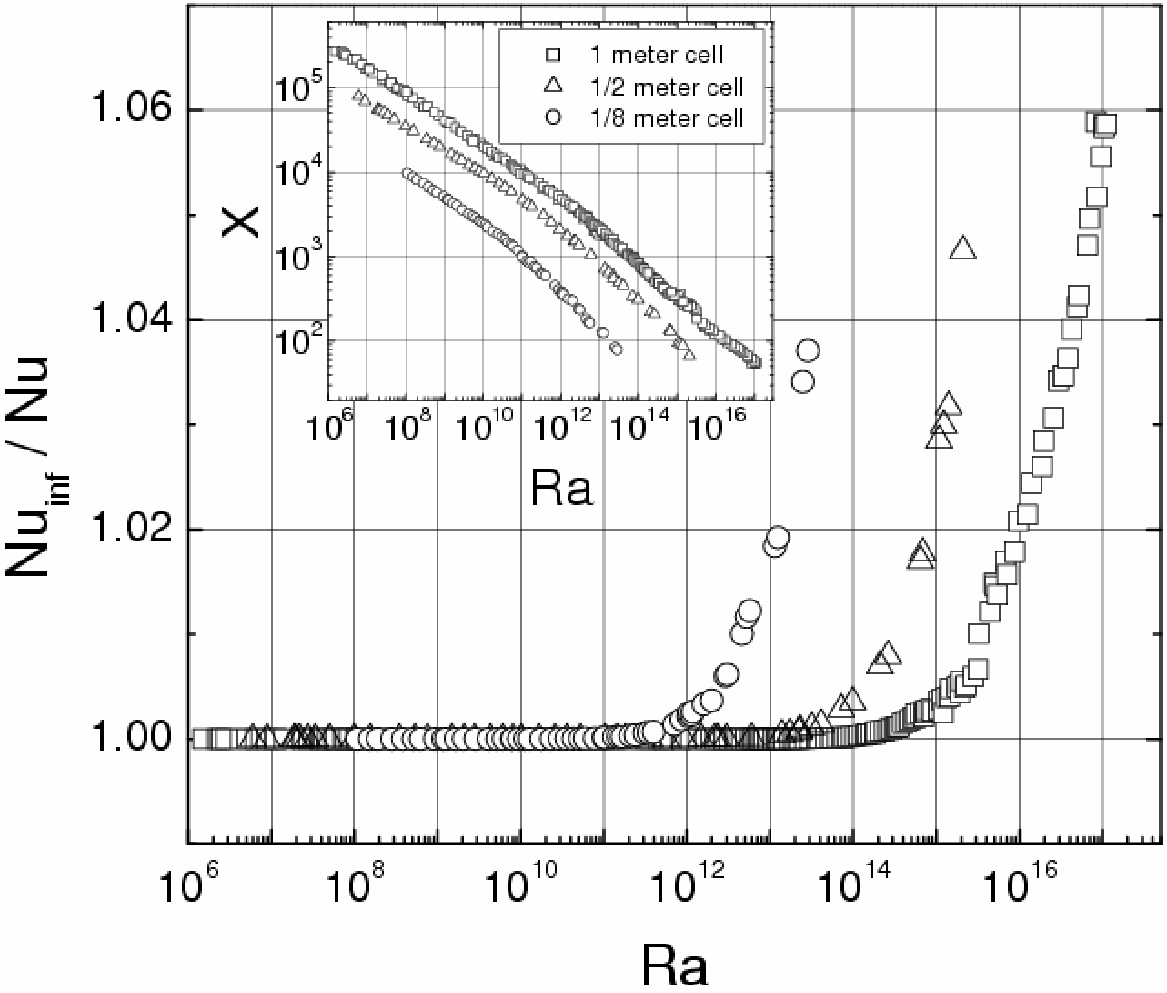
Rotating lattice



side views (data)

top views (schematic)

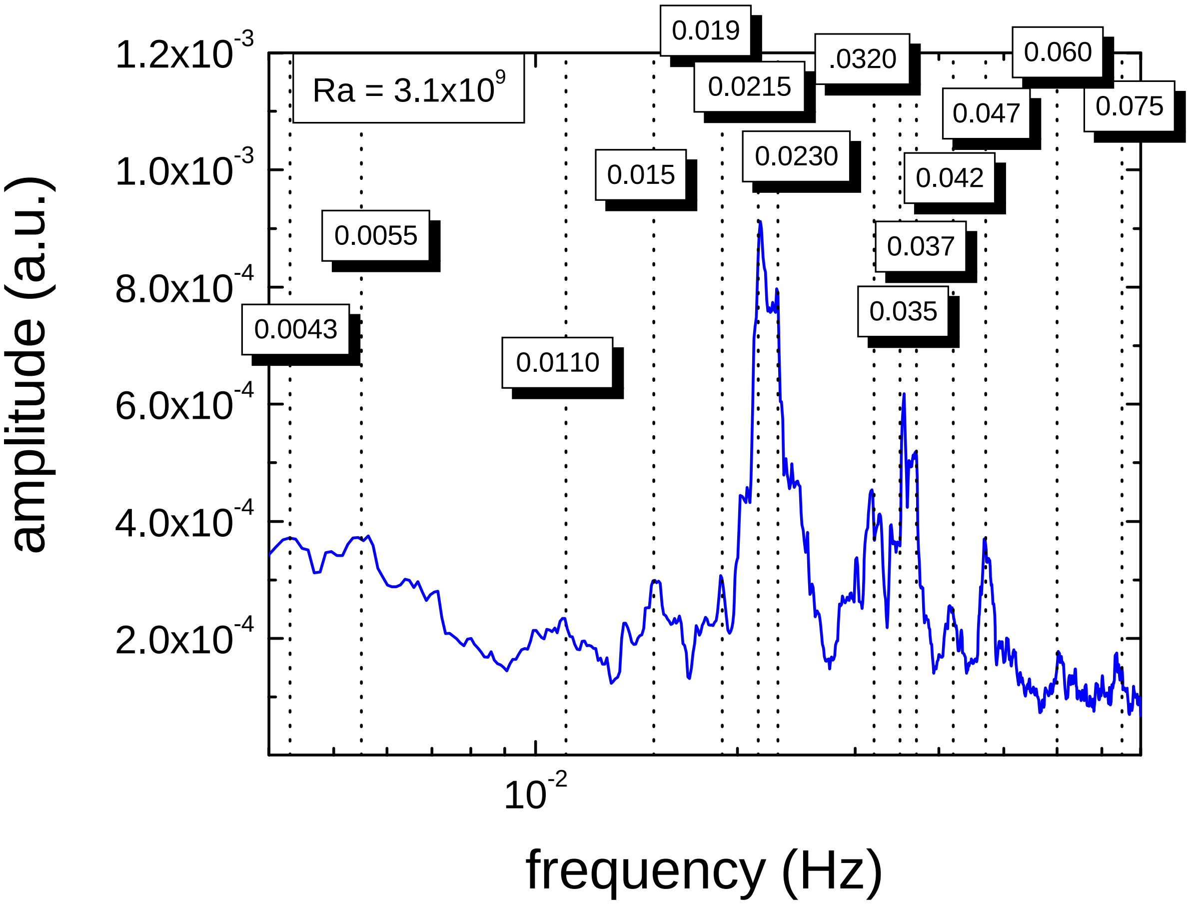




a correction for finite thermal conductivity

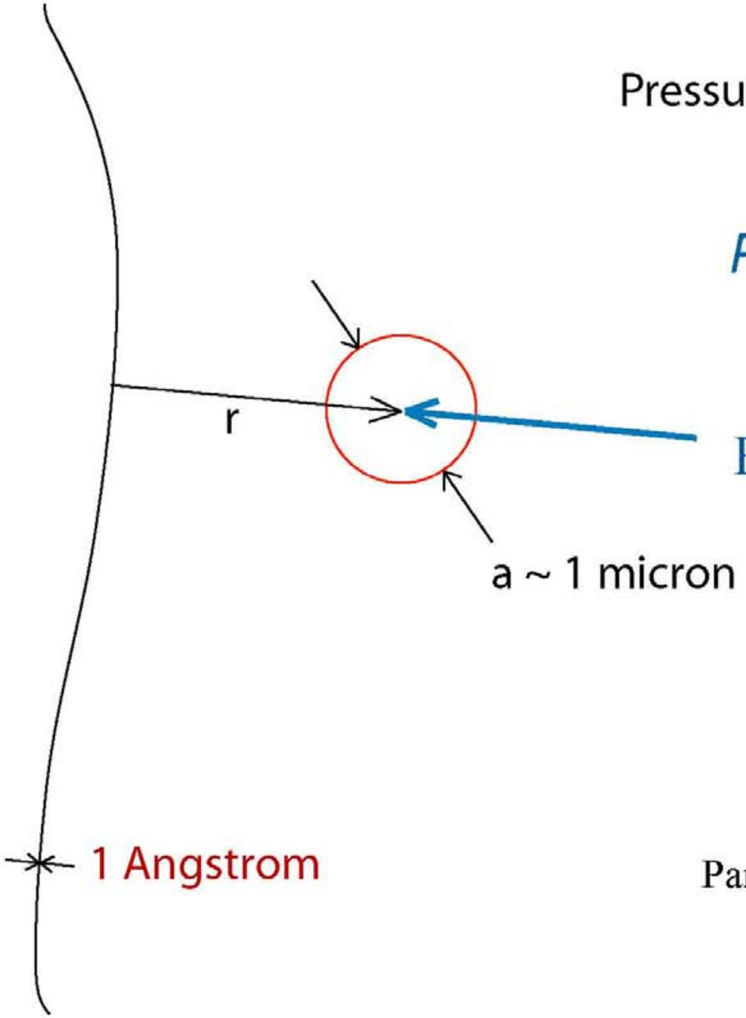
the spectrum of the temperature near the sidewall

aspect ratio 4 and Ra = 3.1 x 10^9



0.021 = f_1
0.037 = f_2
0.023= f_3
0.032 = 2(f_2-f_1)
0.047 = 3(f_2 - f_1)
0.042 = 2f_1
0.060 = 3f_1
0.075 = 2f_2
0.015 = f_2 - f_1
0.0055 = 2f_1 -f_2
0.011 = 4f_1 - 2f_2
0.0043 = 3f_2 – 5f_1
0.019= 2f_1-f_3

Particle Trapping



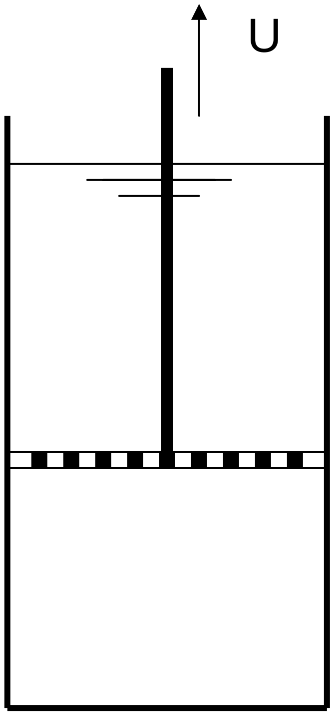
Pressure gradient:

$$P = \frac{-\rho_s \kappa^2}{8\pi^2 r^2}$$

$$F = \frac{4}{3}\pi a^3 \nabla P \sim \frac{1}{r^3}$$

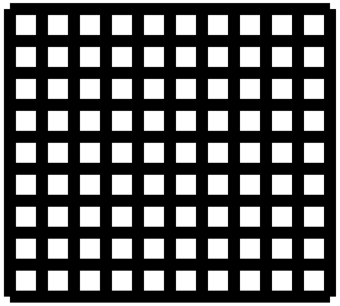
Parks, P.E. and Donnelly, R.J. (1966), *Phys. Rev. Lett.* **16**, 45–48.

What happens if we pull a grid through helium II?

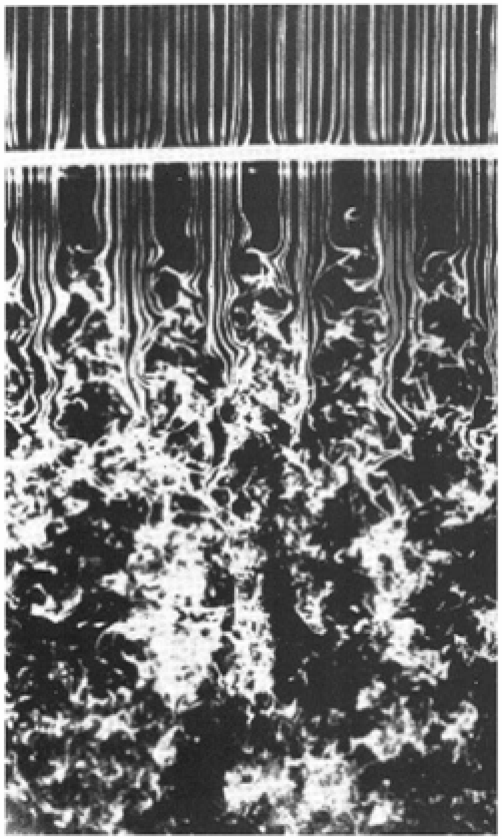


tank of water

nearly isotropic
turbulence is
generated.

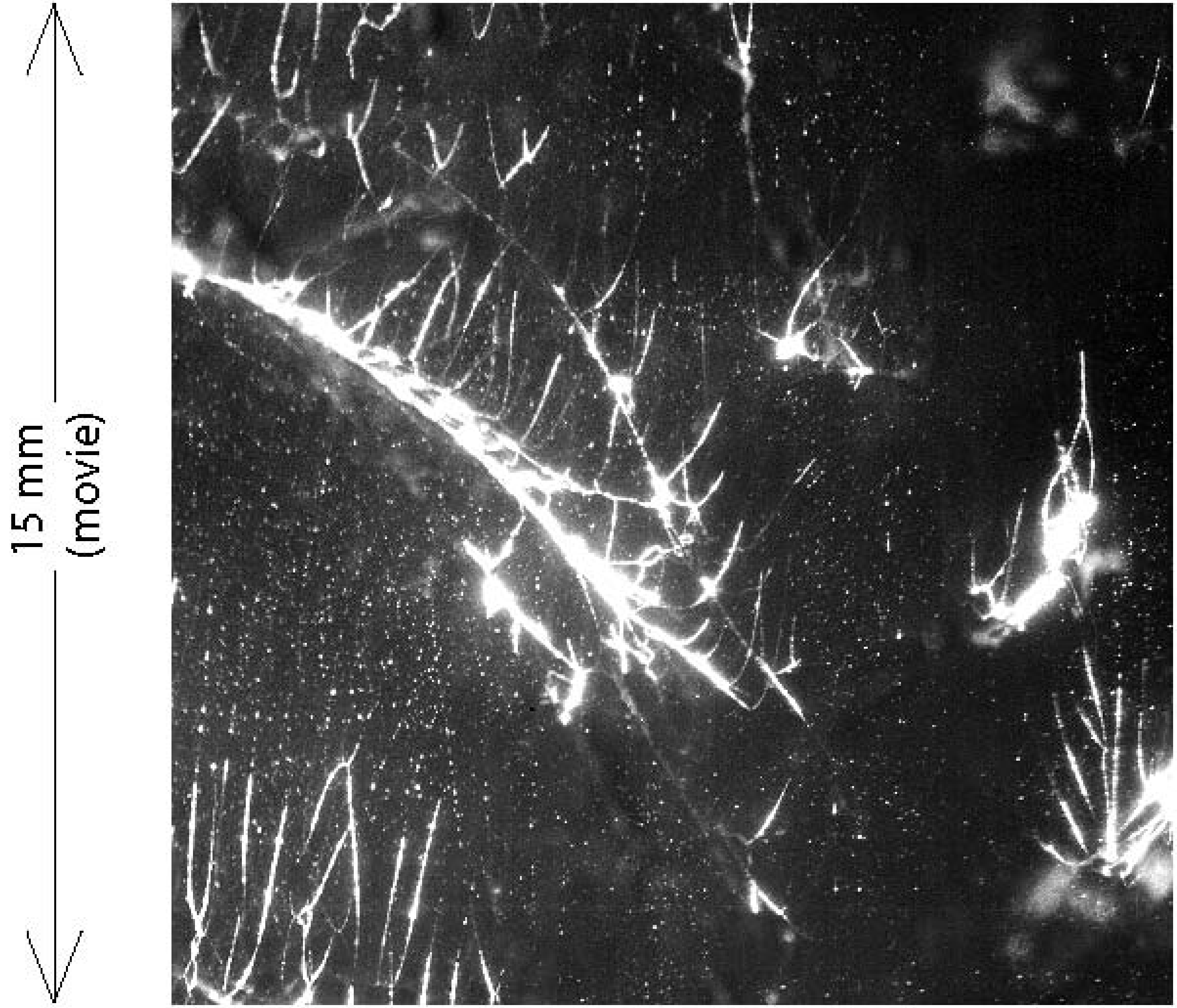


square grid of bars



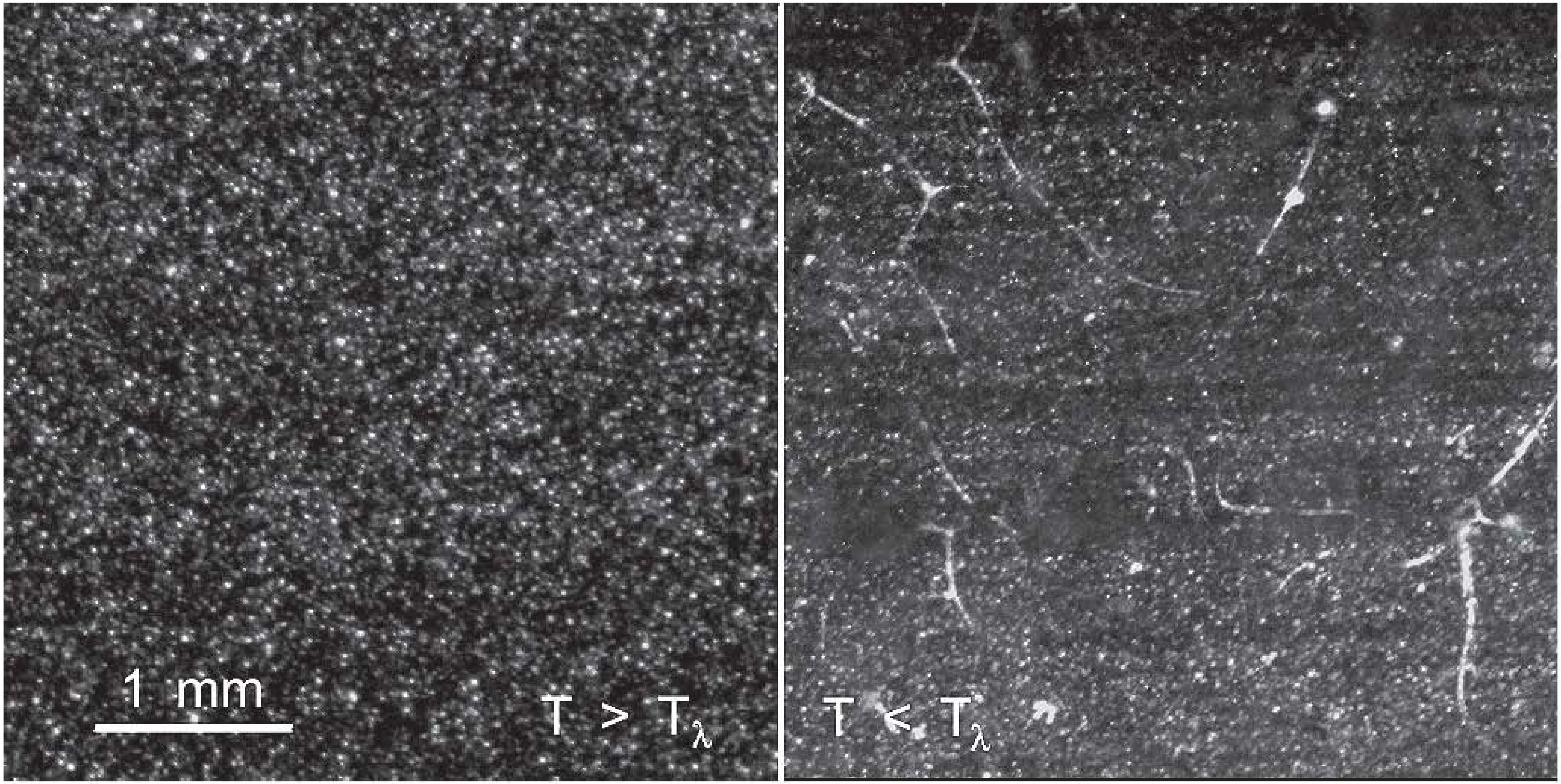
grid turbulence in air:
reoriented; Corke & Nagib

Rotating superfluid

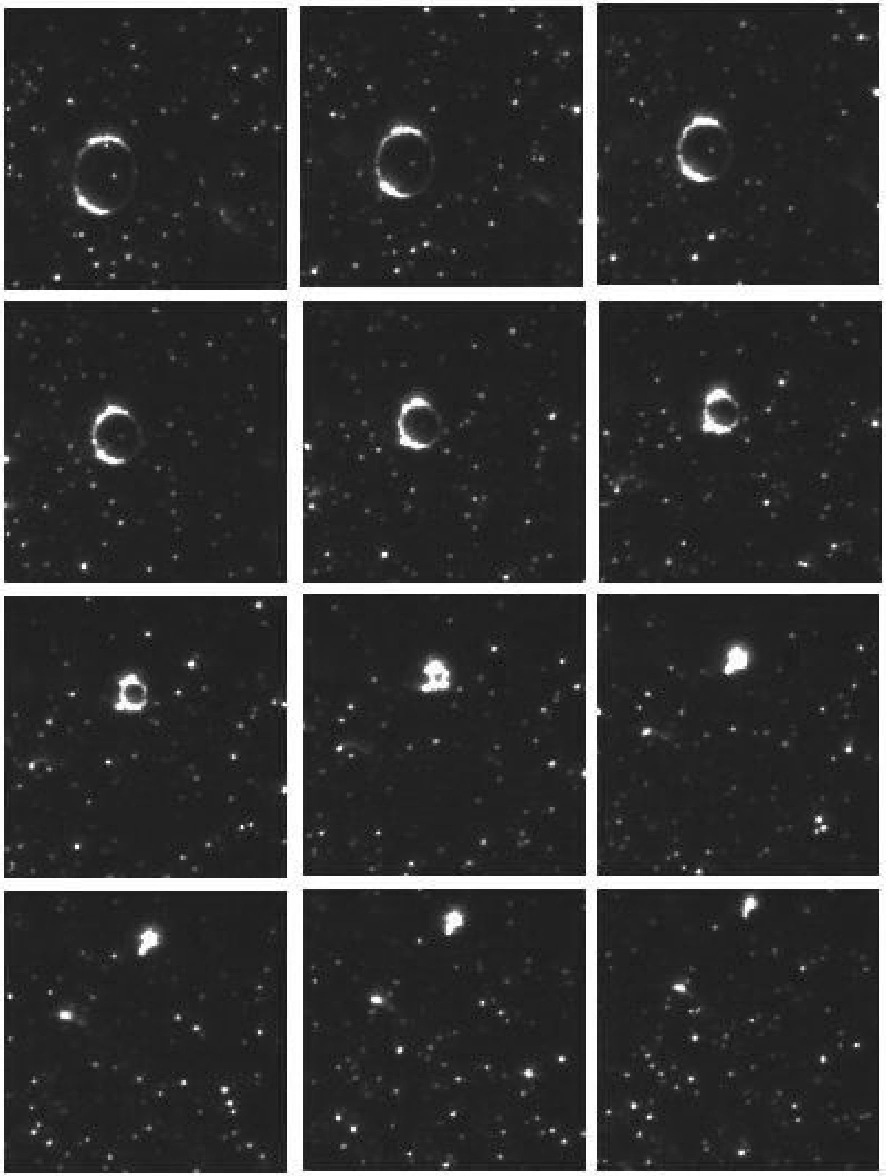


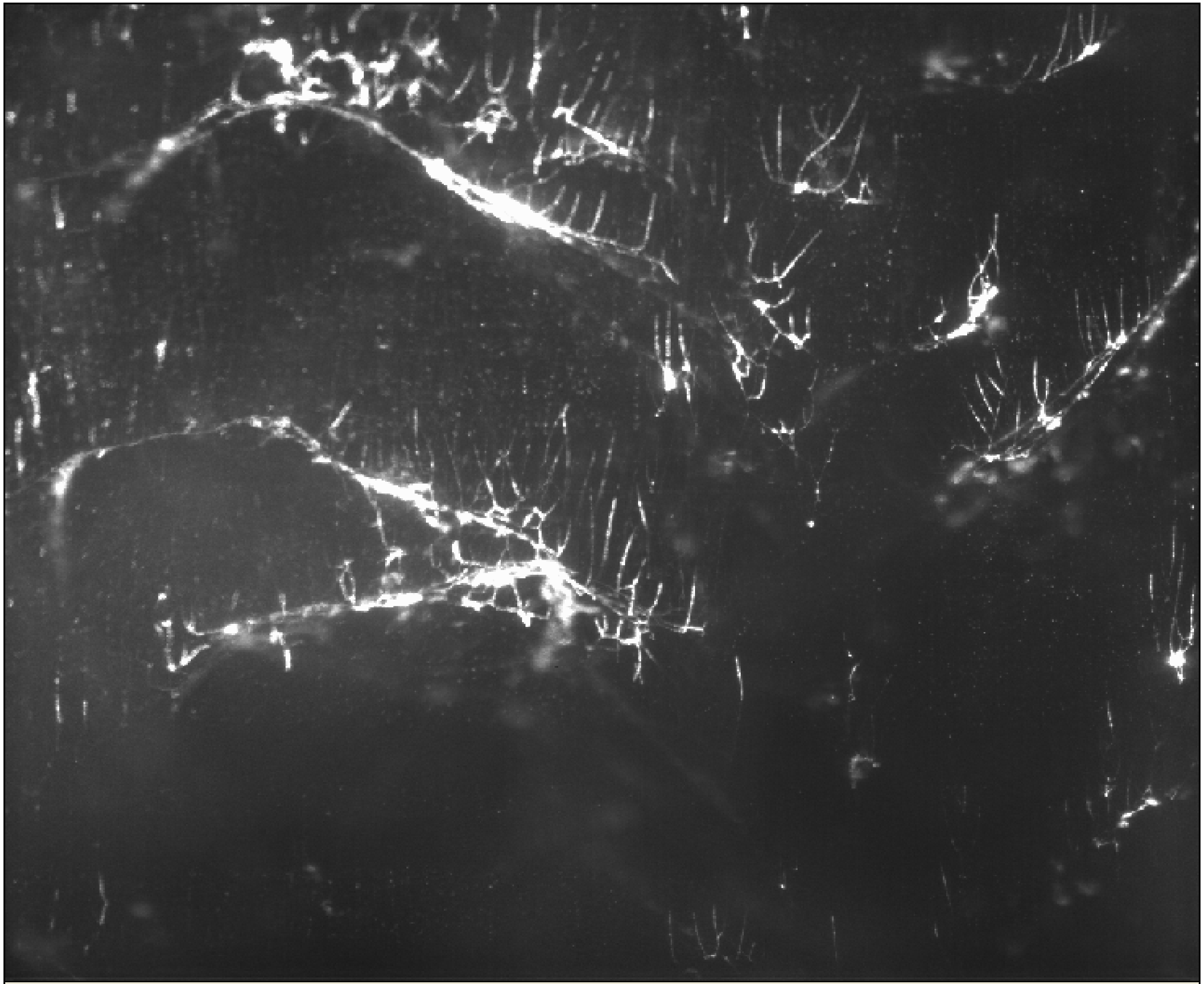
50 years later…

Filaments in Superfluid Helium



G.P. Bewley, D.P. Lathrop & K.R. Sreenivasan, *Nature* 441, 558 (2006);
also Experiments in Fluids (submitted, 2006)





DEFECTS

It is reasonable to assume that Λ = constant (since Λ grows with time and will, at some point, be limited by the channel width)

Consequences

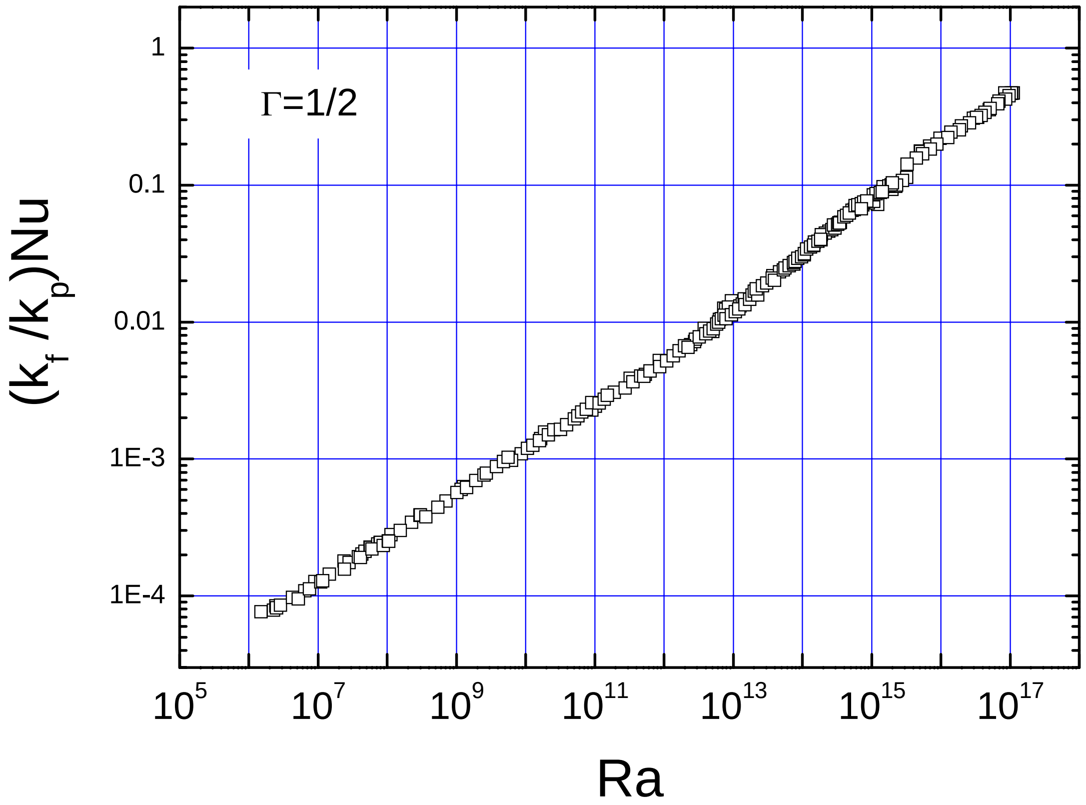
1. $\langle\varepsilon\rangle = d^2t^{-3}$

2. Using the exact relation between enstrophy and dissipation,

$$\langle\varepsilon\rangle = \nu\langle\omega^2\rangle,$$

we obtain

$$\langle\omega^2\rangle^{1/2} = (d/\nu^{1/2})\, t^{-3/2}.$$



effective thermal conductivity of the helium gas as a function of Ra

S.S. Penner
Ph.D. in 1946; at UCSD since 1964

Inaugural Lecture by S.S. Penner: January 24, 2000

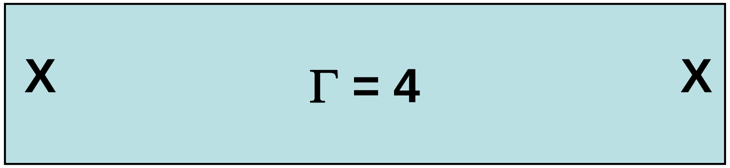
“Energy Supplies for the Twenty-First Century”

The lecture dealt with the following topics:

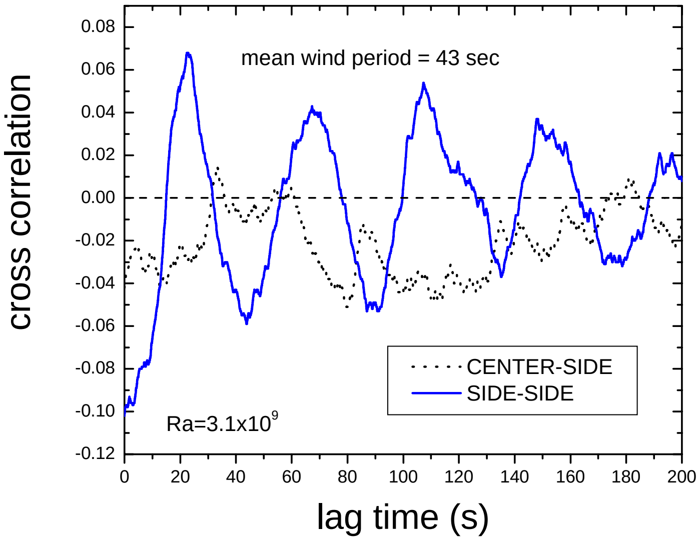
(1) How much energy will be needed for 10 billion people in 2050 and later to ensure a desirable standard of living for all?
(2) How extensive are our known non-renewable and renewable energy resources and reserves?
(3) How long will they last while meeting the world's estimated needs?
(4) What are our options if we terminate the use of fossil fuels in order to eliminate man-made contributions to the concentration of carbon dioxide in the atmosphere while securing worldwide economic well being?
(5) What are our options if we terminate the use of nuclear fission and breeder reactors in order to stop the accumulation of long-lived radioactive isotopes and reduce the risks of nuclear proliferation while securing worldwide economic well being?
(6) What are our options if we interdict simultaneously the use of fossil fuels and of nuclear fission reactors while securing worldwide economic stability?
(7) What research and development programs (other than the construction of fusion reactors) should we emphasize while aiming for a stable world order without using fossil fuels and nuclear fission or breeder reactors?

What about higher aspect ratio?

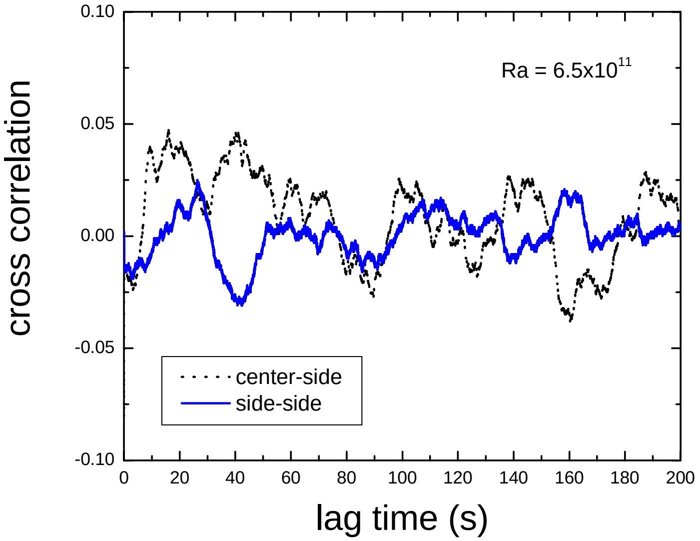
Niemela & Sreenivasan, *J. Fluid Mech.* 557, 411 (2006)



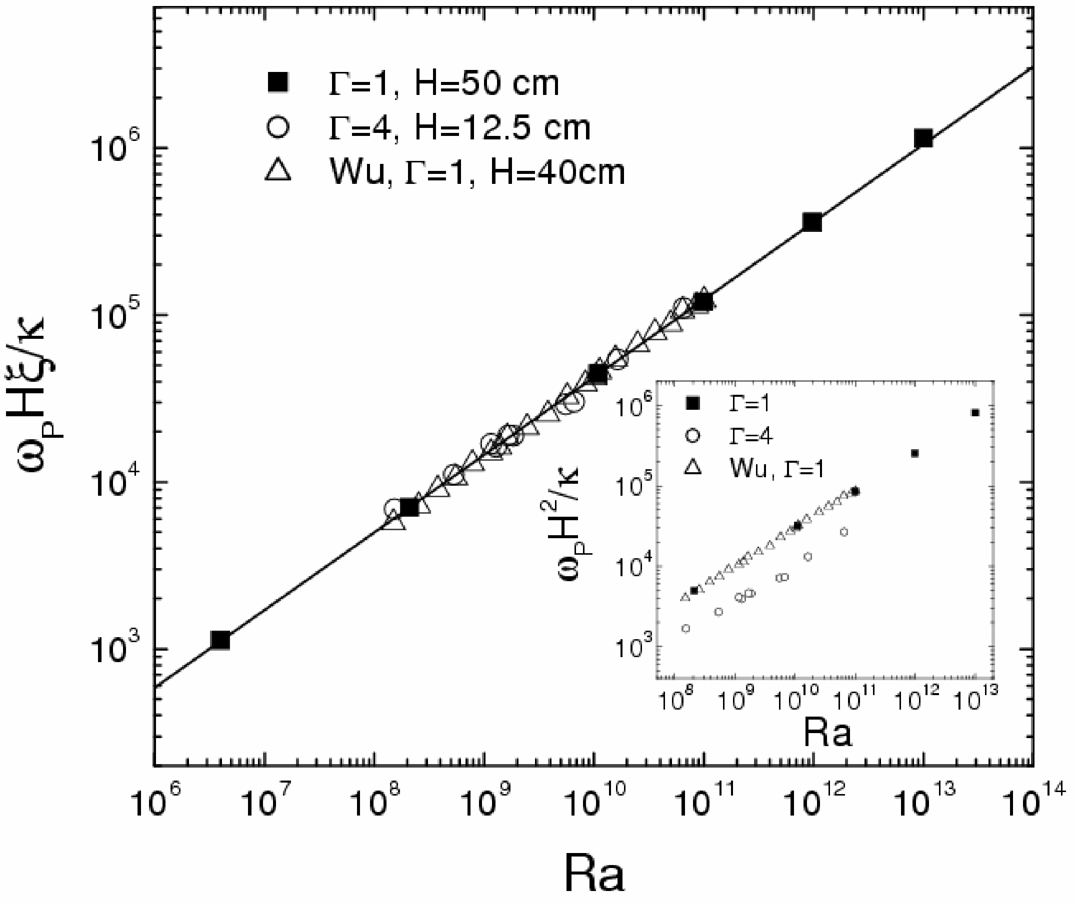
correlation of signals from opposite sides and that between the side and center



Ra < 3 x 10^{10}

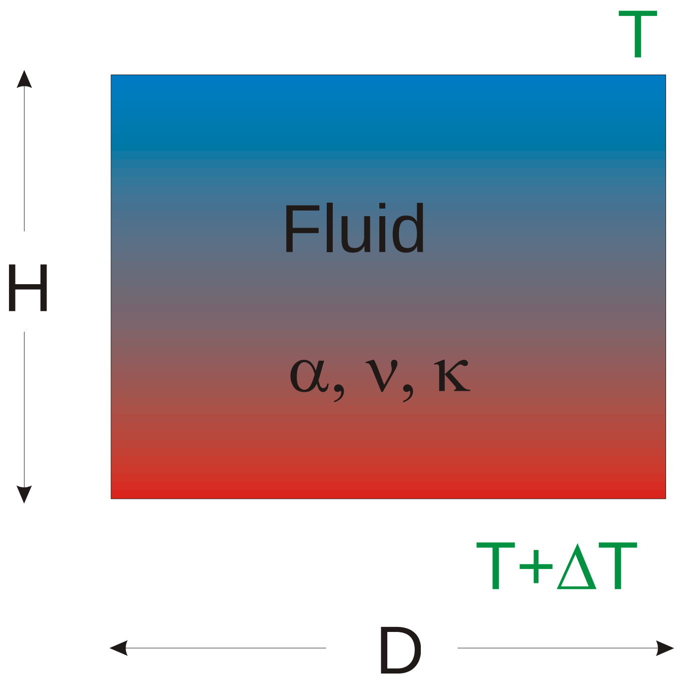


Ra > 3 x 10^{10}



scaling of the wind frequency for different aspect ratios

Thermal Convection using Classical Helium



T

Fluid

α, ν, κ

H

T+ΔT

D

control parameters for convection

$$\mathrm{Ra} = \frac{g\alpha\Delta T H^3}{\nu\kappa}$$

$\mathrm{Pr} = \nu/\kappa$

$\Gamma = D/H$

S ~ detailed shape ??

the cryogenic apparatus for very high Ra