Heavy Ions Collisions and the search for the Quark-Gluon Plasma

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1. QCD matter. The final state in HIC.
2. The first stages and before. The initial state in HIC.
3. Signals for the QGP formation.
4. Experimental/Theoretical status.
   - ⇛ RHIC
   - ⇛ LHC

Summary I

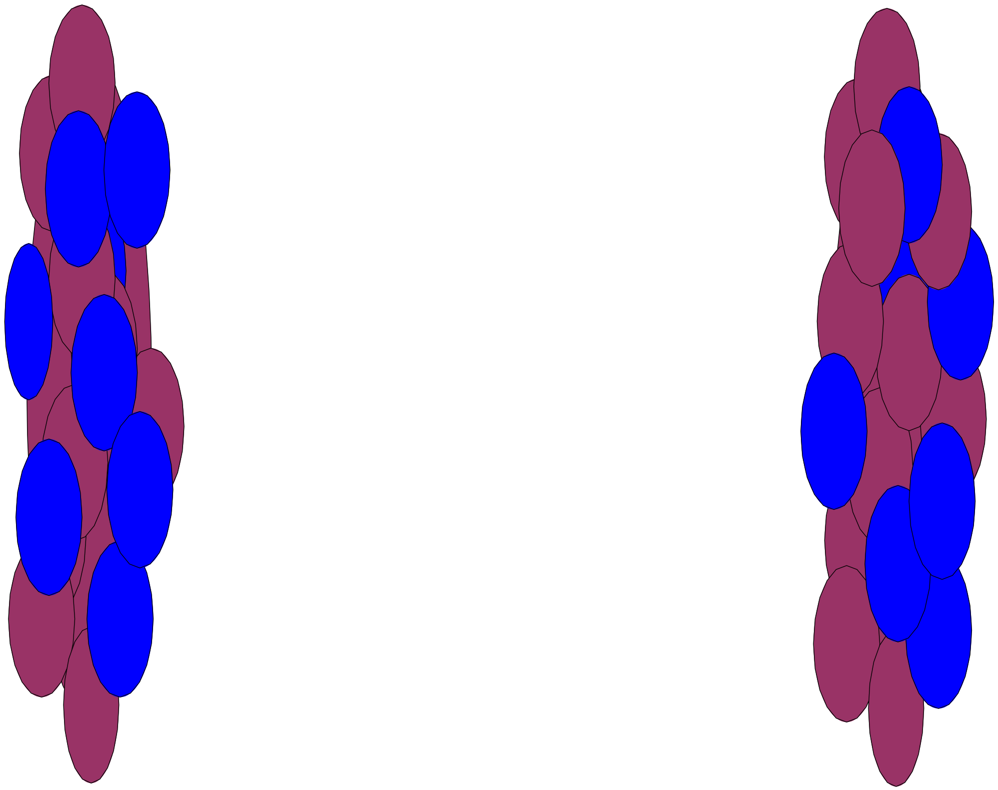
- QCD vacuum:
  Confinement & chiral symmetry breaking
- Other states of matter possible?
- Theory $\longrightarrow$ Different phases exist!

(for small μ_B)
Lattice + perturbative + models

- Transition hadron gas $\leftrightarrow$ quark gluon plasma.
- Order of the transition depends on quarks masses. For realistic masses, most probably crossover at $\mu_B = 0$.
- Increasing $\mu_B \longrightarrow$ first order phase transition.
- Critical point (second order phase transition).
- Heavy ion collisions experiments attempt to study this region.

2. Initial state

High-energy heavy ion collisions



Before the collision: Lorentz-contracted nuclei

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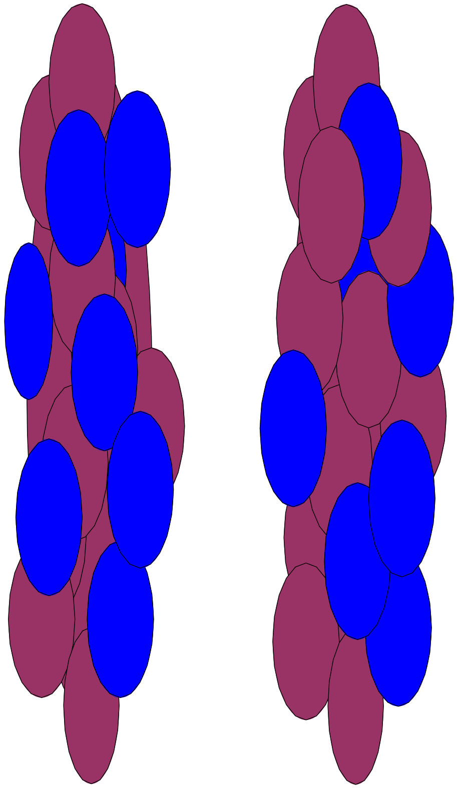

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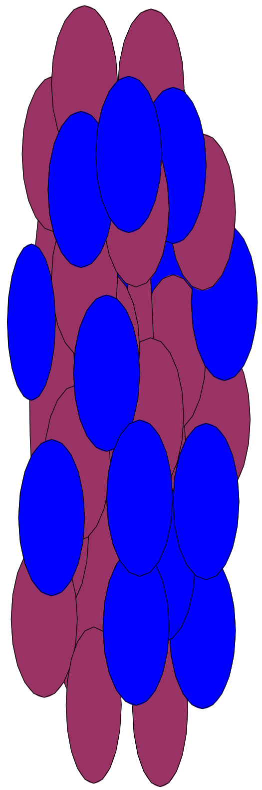
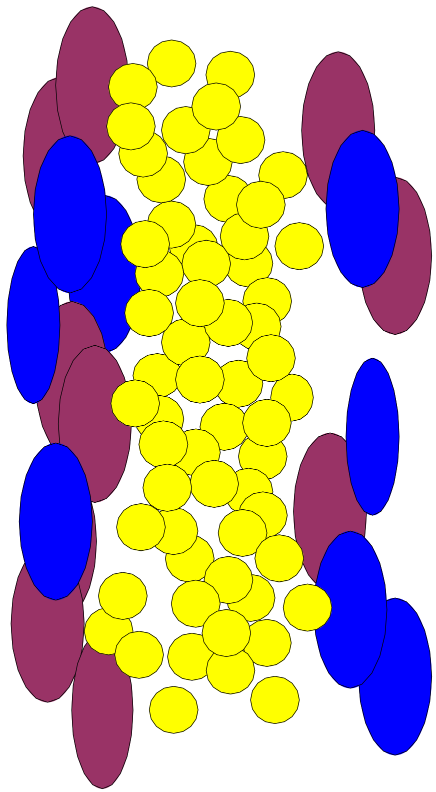


at $t = 0$ most of the energy in the central region

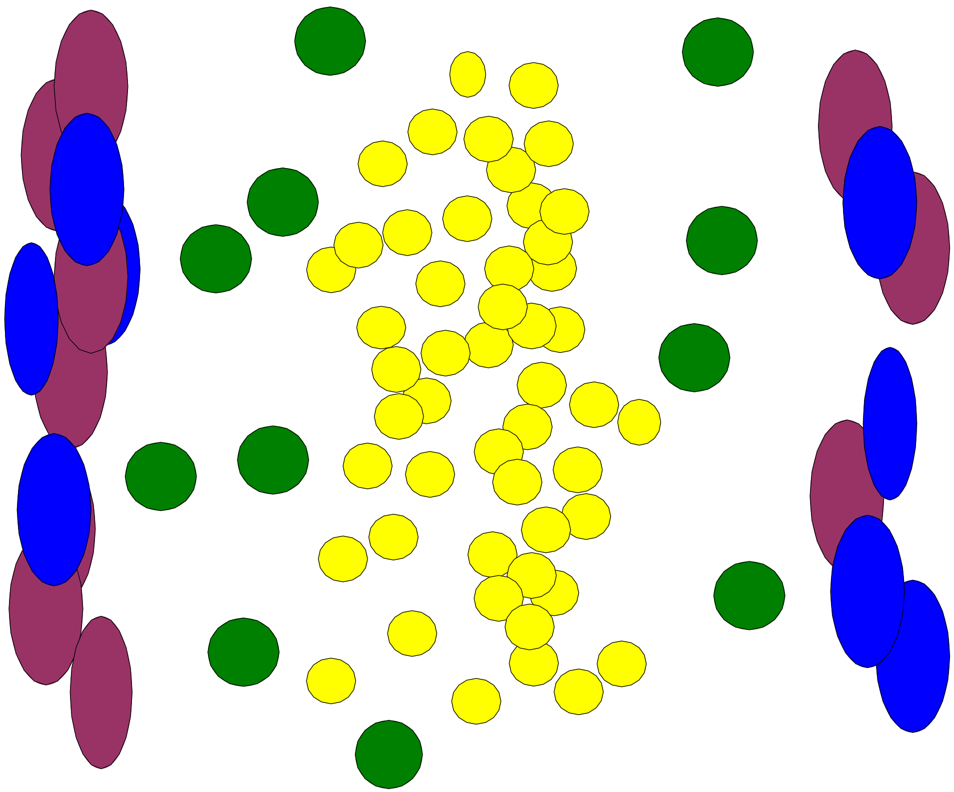
Initial state

High-energy heavy ion collisions



First $\sim 0.1 \div 0.3$ fm. Quark gluon plasma formation

High-energy heavy ion collisions



Expansion and hadronization

High energy heavy ion collisions

Complicated systems: many particles both in the initial and final state

- Initial state:
  - Nuclei composed by nucleons
  - Multiple scattering.
  - High-density effects (saturation).
  - Colliding objects are extended objects.
    - Coherence effects.
    - Space-time picture.
- Final state:
  - Thermalization.
  - Phase transitions.
  - ...

High energy heavy ion collisions

Complicated systems: many particles both in the initial and final state

- Initial state: Study simpler systems $\longrightarrow$ pA
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  - Colliding objects are extended objects.
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    - Space-time picture.
- Final state:
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Some variables

- Light cone variables:

$$x^{\pm} = x_0 \pm x_3 \quad p^{\pm} = E \pm p_3$$

$$p \cdot x = \frac{1}{2}\left(p^+ x^- + p^- x^+\right) - \mathbf{p}_\perp \cdot \mathbf{x}_\perp$$

- Rapidity: $y = \frac{1}{2} \ln \left[\frac{E + p_3}{E - p_3}\right] = \frac{1}{2} \ln \left[\frac{p_+}{p_-}\right]$

  Boost: $y' = y + y_\beta$ where $y_\beta = \frac{1}{2} \ln \left[\frac{1+\beta}{1-\beta}\right] \longrightarrow$ *(additive velocity)*

- Pseudorapidity: $\eta = \ln[\cot(\theta/2)]$; $\eta = y$ for massless particles.

- Spacial rapidity

$$y_{\rm sp} = \frac{1}{2} \ln \left[\frac{x_0 + x_3}{x_0 - x_3}\right] = \frac{1}{2} \ln \left[\frac{x_+}{x_-}\right]$$

  For particles produced at $\mathbf{x} = 0$, $t = 0 \longrightarrow y_{\rm sp} = y$

- Proper time: $\tau = \sqrt{x_0^2 - x_3^2}$

Geometry

Proton-nucleus collision in the nucleus rest frame.

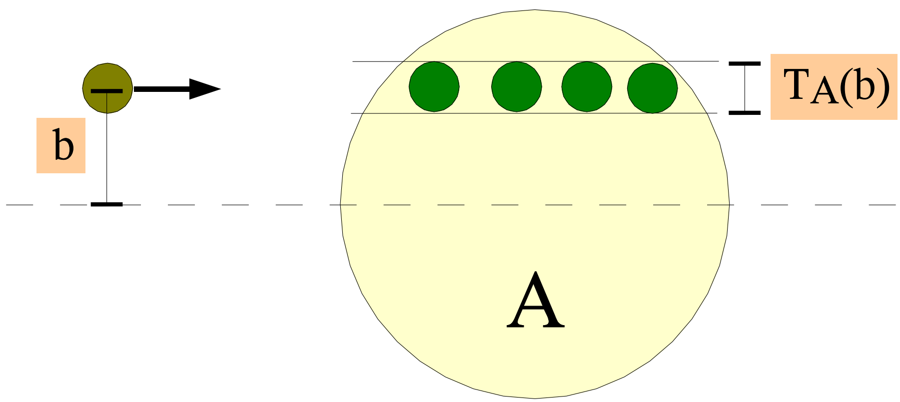
⇒ $\rho(\mathbf{b}, z)$ is the nuclear density. $\int dz d\mathbf{b} \rho(\mathbf{b}, z) = 1$

⇒ $T_A(\mathbf{b})$ is the profile function (amount of nuclear matter in the transverse plane.)

$$T_A(\mathbf{b}) = \int_{-\infty}^{\infty} dz \rho(\mathbf{b}, z)$$

⇒ b – impact parameter

Centrality



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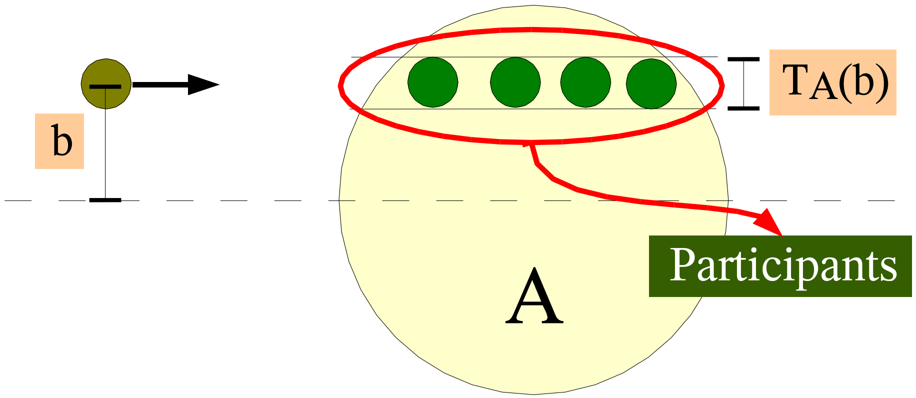


- b – impact parameter
  Centrality
- Number of nucleons per $\mathbf{b}$

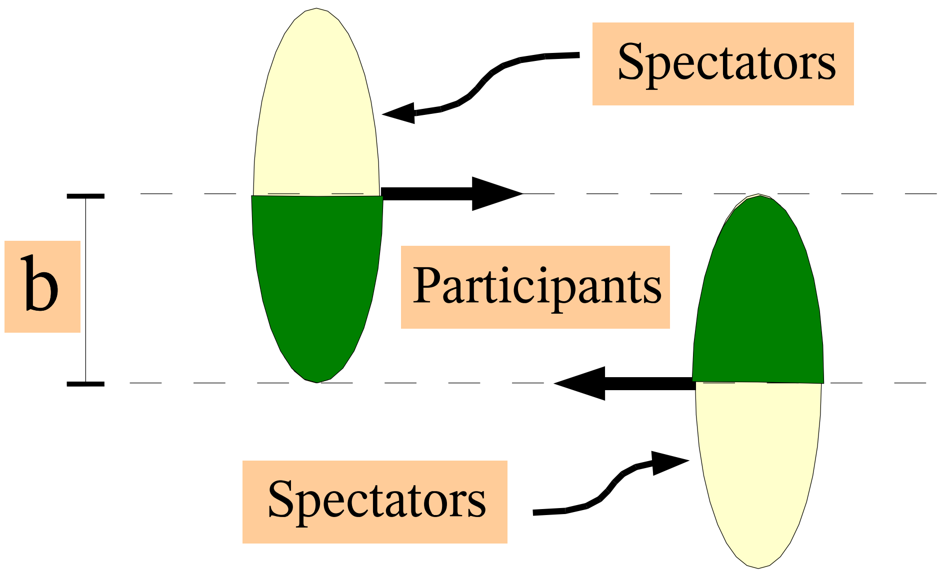
$$AT_A(\mathbf{b})$$

- Number of NN collisions: hit nucleons Participants

$$N_A(\mathbf{b}) = AT_A(\mathbf{b})\sigma_{NN}$$

Geometry II

Nucleus-Nucleus collisions in the center of mass frame.



Number of elementary NN collisions

$$N_{coll}(b) = ABT_{AB}(b)\sigma_{NN}$$

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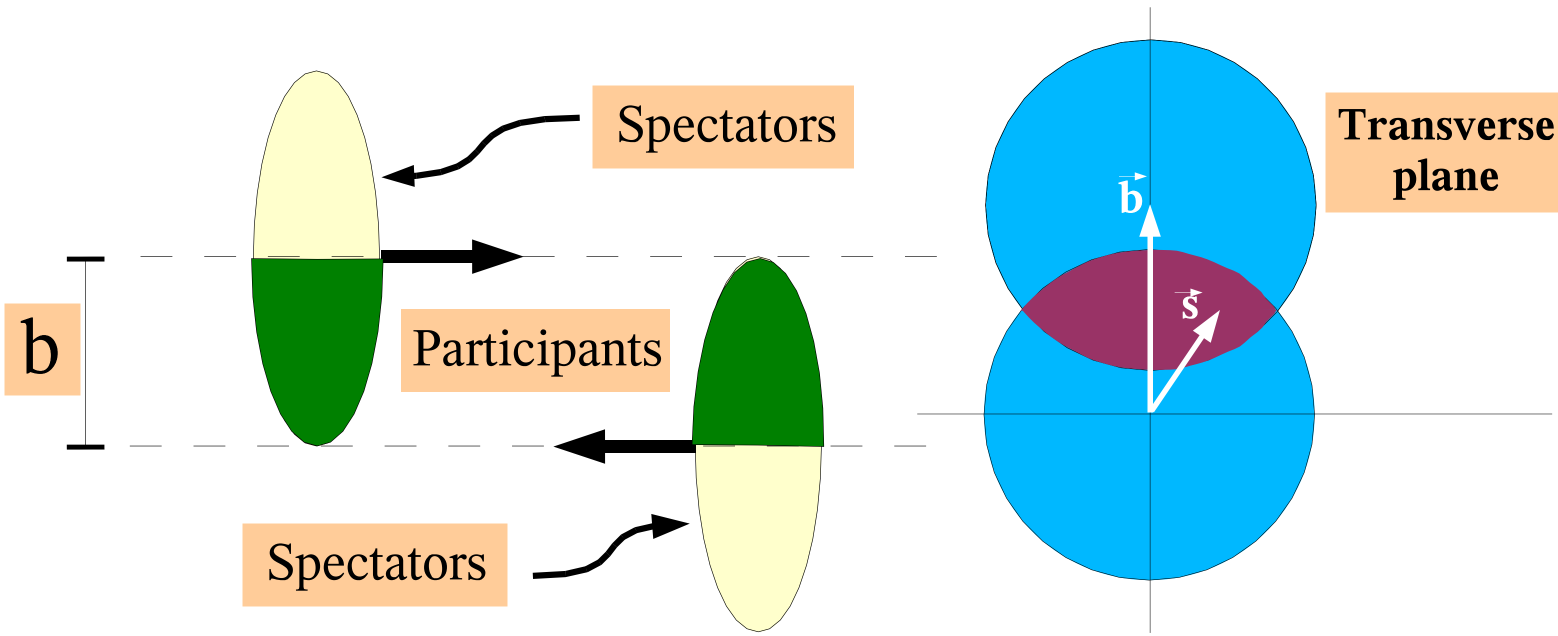


Number of elementary NN collisions

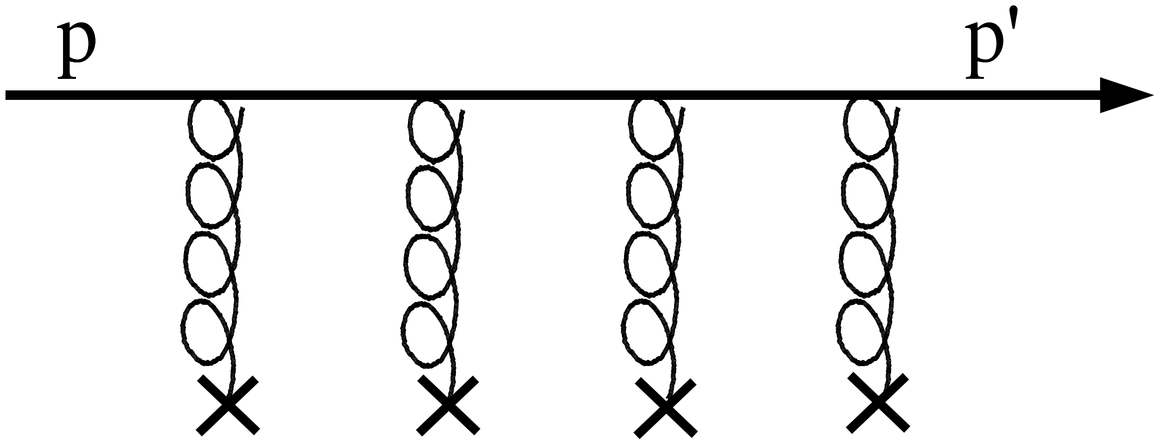
$$N_{coll}(b) = ABT_{AB}(b)\sigma_{NN}$$

Now, the transverse plane is more involved:

$$T_{AB}(\mathbf{b}) = \int d^2\mathbf{s} T_A(\mathbf{s}) T_B(\mathbf{b}-\mathbf{s})$$

Multiple scattering

Example: scattering with potentials. General structure (scalar quark)



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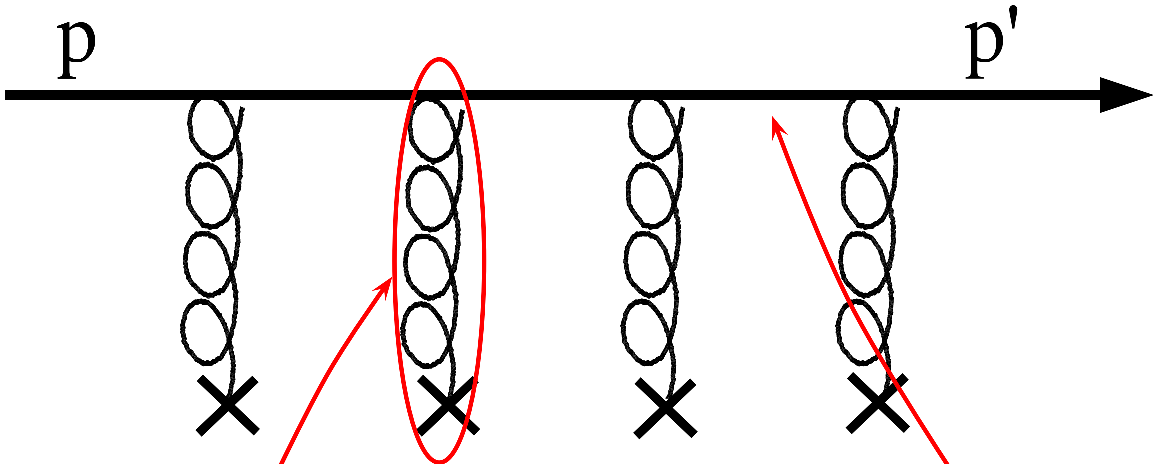

$$\int d^4x A^\mu(x) \exp\{ix(k_i - k_{i+1})\}$$

$$\frac{1}{k_i^2 + i\epsilon}$$

Potentials

Propagators

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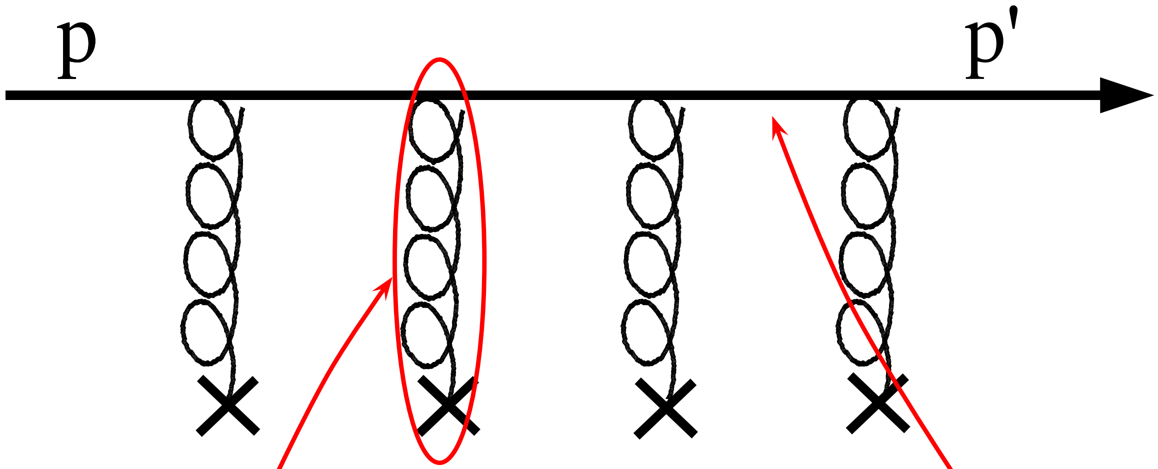

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High-energy limit (eikonal approximation) $p_+ \to \infty$, $p_- \sim 0$
$\longrightarrow$ the integrals in dk_- can be done. Coherence factors

$$\exp\{i\frac{p_\perp^2}{2p_+}\Delta z\} \quad \Longleftrightarrow \quad \text{coherence length } \; l_{\rm coh} \sim \frac{2p_+}{p_\perp^2}$$

Example: 2 scatterings

The contribution of two scatterings to the S-matrix is

$$S_2(p',p) = \int \frac{d^4k}{(2\pi)^4} \left\{ -ig(k_\mu + p'_\mu) \int d^4x_2 A^\mu(x_2) \mathrm{e}^{ix_2(p'-k)} \right\}$$
$$\frac{i}{k^2 + i\epsilon} \left\{ -ig(p_\mu + k_\mu) \int d^4x_1 A^\mu(x_1) \mathrm{e}^{ix_1(k-p)} \right\}$$

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The high-energy limit ($p_+ \to \infty$) gives

$$S_2 = 2\pi\delta(p'_+ - p_+) 2p_+ \int \frac{d^2\mathbf{k}_\perp}{(2\pi)^2} \int d^3\mathbf{x_1} \left[-igA_-(\mathbf{x_1}) \right] \int d^3\mathbf{x_2} \left[-igA_-(\mathbf{x_2}) \right] \times$$
$$\times \exp\left\{ i\mathbf{k}_\perp(\mathbf{x_{1\perp}} - \mathbf{x_{2\perp}}) \right\} \exp\left\{ i(\mathbf{x_{2\perp}}\mathbf{p_{2\perp}} - \mathbf{x_{1\perp}}\mathbf{p_{1\perp}}) \right\} \exp\left\{ i\frac{{\mathbf{k}_\perp}^2}{p_+}(x_{1+} - x_{2+}) \right\}$$

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Coherence factor $\Rightarrow$ coherence length $l_{\mathrm{coh}} \sim 2p_+/k_\perp^2$

- $l_{\mathrm{coh}} \gg$ size of the system $\Rightarrow$ totally coherent.
- $l_{\mathrm{coh}} \ll$ size of the system $\Rightarrow S_2 \to 0$ only one scattering contribution survives: totally incoherent $\to$ independent scatterings.

Arbitrary number of scatterings

⇛ In the totally coherent limit, the contributions of an arbitrary number of scatterings can be summed and the S-matrix is

$$S = \int d^2\mathbf{x}_\perp \mathrm{e}^{-i\mathbf{x}_\perp(\mathbf{p}'_\perp - \mathbf{p}_\perp)} \; P \exp\left\{-\frac{ig}{2}\int dx_+ A_-(x_+, x_\perp)\right\}$$

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- The total cross section is given by the optical theorem $\sigma^{\text{tot}} = 2ImA$. Suppose A_- is real

$$\sigma^{\text{tot}} = 2\left(1 - \mathrm{e}^{-\sigma^{\text{tot}}_{\text{sing}}/2}\right)$$

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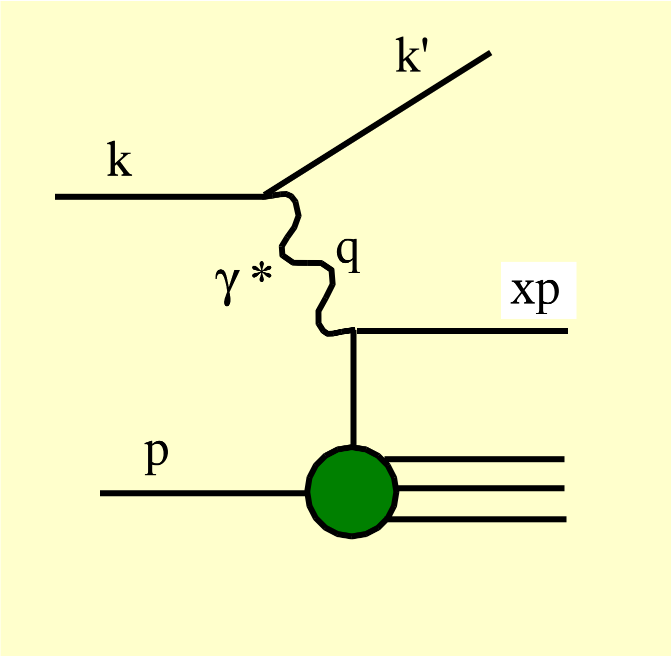
$$\sigma^{\mathrm{tot}} = 2\left(1 - \mathrm{e}^{-N\sigma/2}\right) \xrightarrow[\text{incoherent}]{} \sigma^{\mathrm{tot}} = N\sigma$$

Coherence $\longrightarrow$ suppression

Structure of the nucleus

Deep inelastic scattering



- Virtuality: $Q^2 \equiv -q^2$
  For large $Q^2 \Rightarrow \alpha_s(Q^2)$ pQCD.
- Bjorken x

$$x = \frac{Q^2}{2p \cdot q}$$

  Fraction of momentum of the hadron carried by the parton (quark)
- Cross section (LO)

$$\sigma^{\gamma^* p}(s, Q^2) = \frac{4\pi^2 \alpha_{\rm em}}{Q^2} F_2(x, Q^2)$$

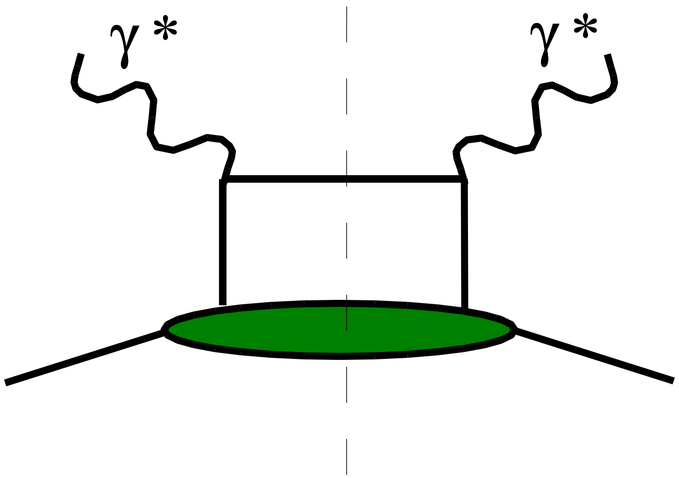
The structure function is

$$F_2(x, Q^2) = \frac{4}{9}\left[u(x, Q^2) + \bar{u}(x, Q^2)\right] + \frac{1}{9}\left[d(x, Q^2) + \bar{d}(x, Q^2)\right] + \frac{1}{9} s(x, Q^2) + \ldots$$

The parton distribution functions (PDF) cannot be computed from pQCD, only its evolution in Q^2

Evolution equations

- DIS measures the quark content of the proton



Evolution equations

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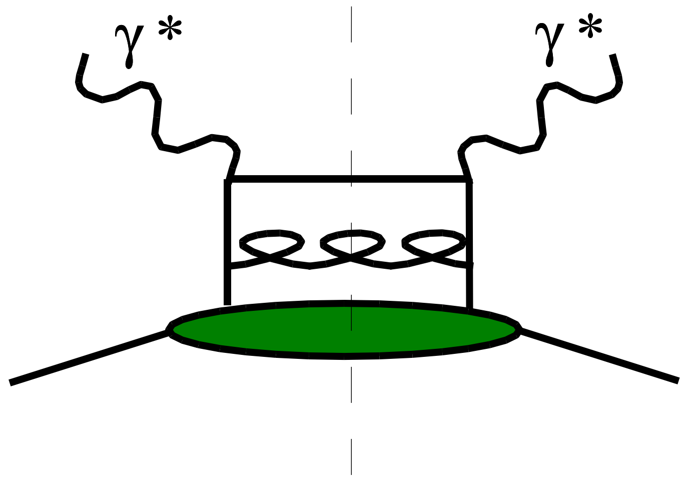
- Quantum corrections ex.

$\rightarrow 1/k_t^2$ singularities



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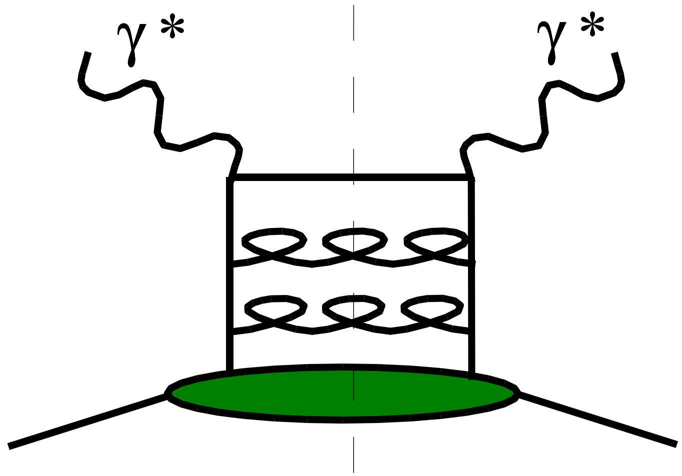

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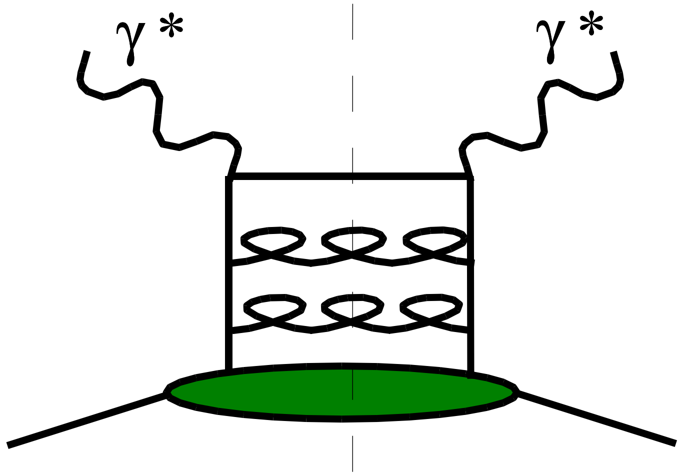


[Dokshitzer, Gribov, Lipatov, Altarelli, Parisi]

$$\frac{\partial q(x,Q^2)}{\partial \log Q^2} = \frac{\alpha_s(Q^2)}{2\pi}\left(P_{qq}\otimes q + P_{qg}\otimes g\right)$$

$$\frac{\partial g(x,Q^2)}{\partial \log Q^2} = \frac{\alpha_s(Q^2)}{2\pi}\left(P_{gq}\otimes q + P_{gg}\otimes g\right)$$

where

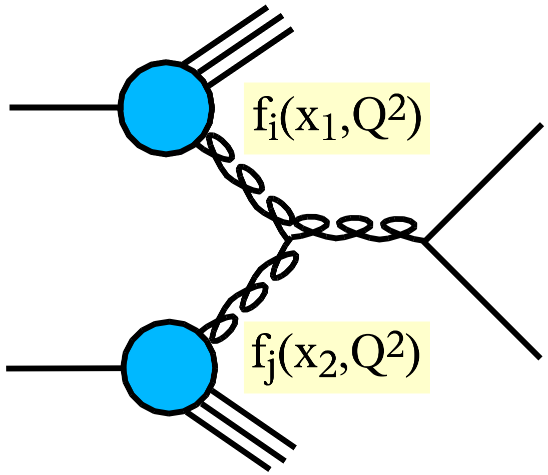
$$P_{ij}\otimes f_j \equiv \int_x^1 \frac{dy}{y} P_{ij}\left(\frac{x}{y}\right) f_j(y,Q^2)$$

Factorization formula

Factorization $\longrightarrow$ PDF's universal. The same for all processes. Cross sections

$$\sigma_{pp}^{X} = \sum_{ij} f_i^p(x_1, Q^2) \otimes f_j^p(x_2, Q^2) \otimes \sigma^{ij \to X}$$

Example:



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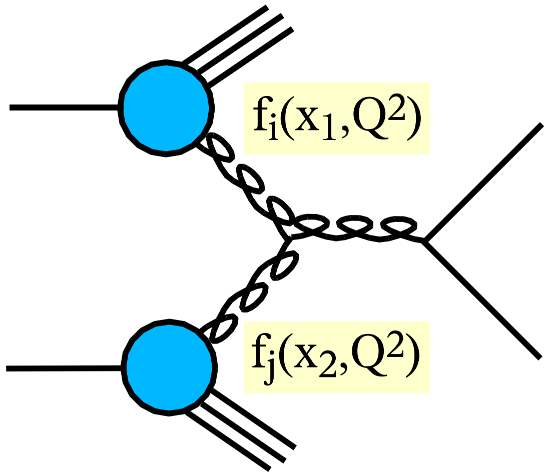

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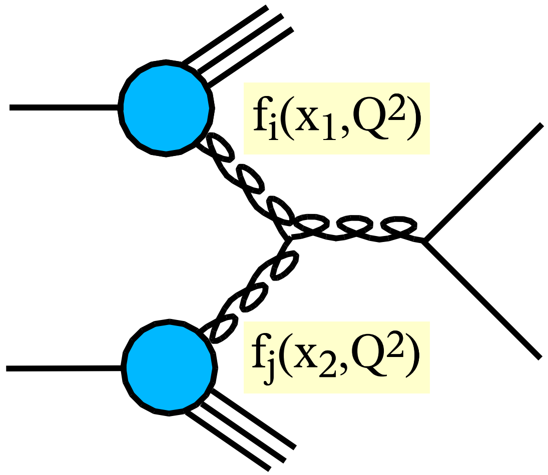
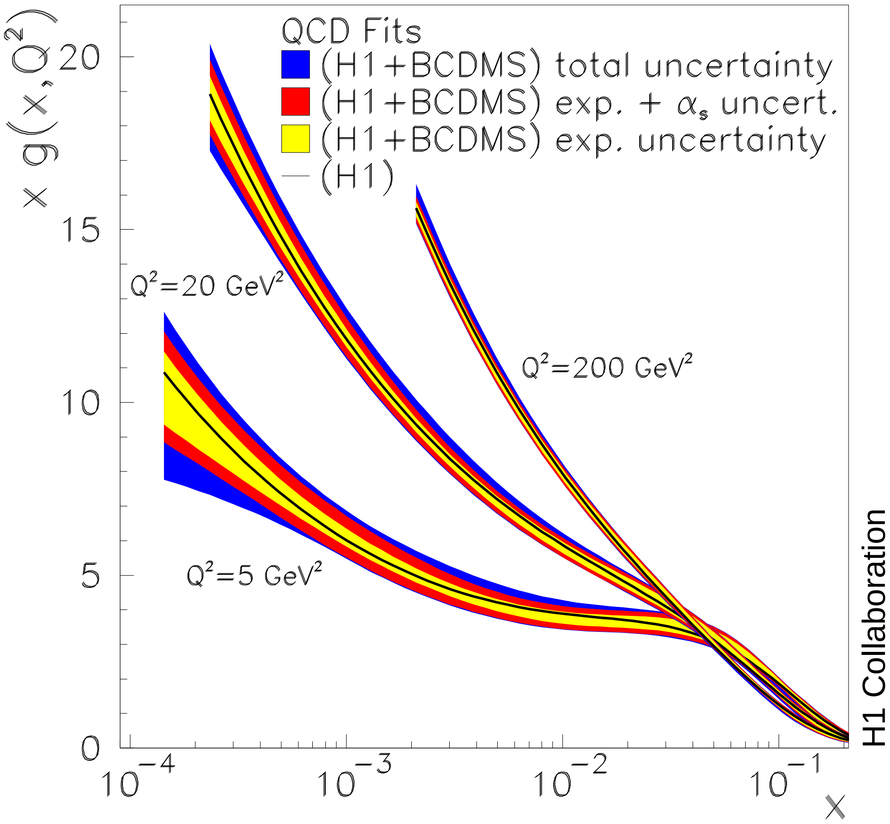


- At LO PDF $\longrightarrow$ probability to find a parton inside the proton with fraction of momentum x for a given virtuality.
- Using the factorization formula, the PDF of the proton are obtained from fits to experimental data.

Parton distributions in the proton

Gluon distribution inside the proton



gluon distribution grows very fast at small-x

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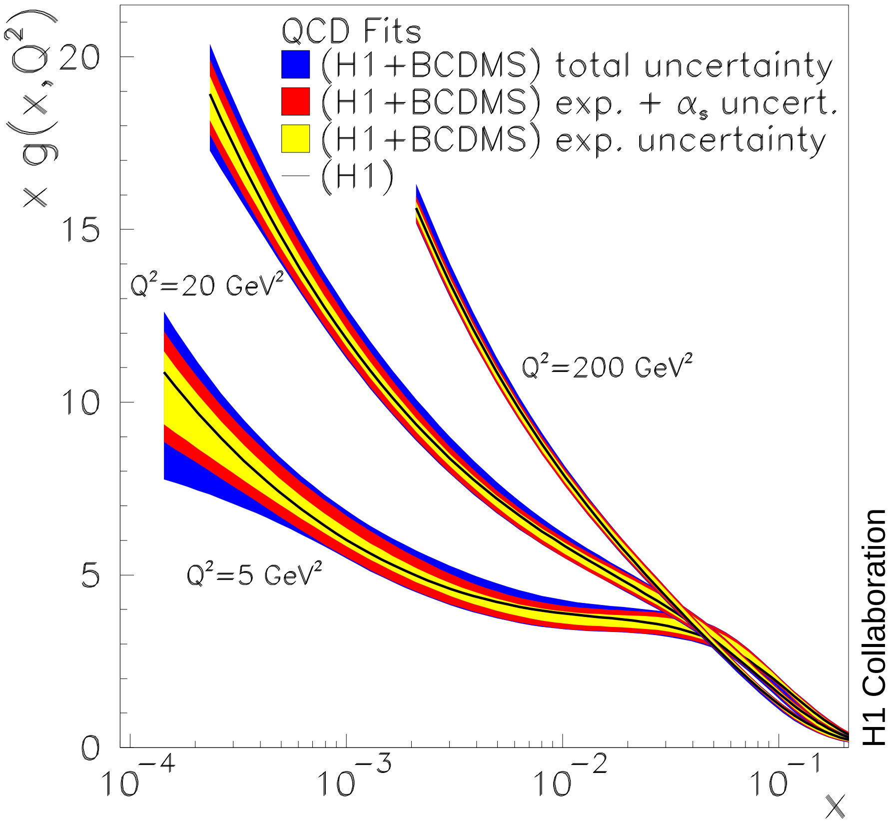
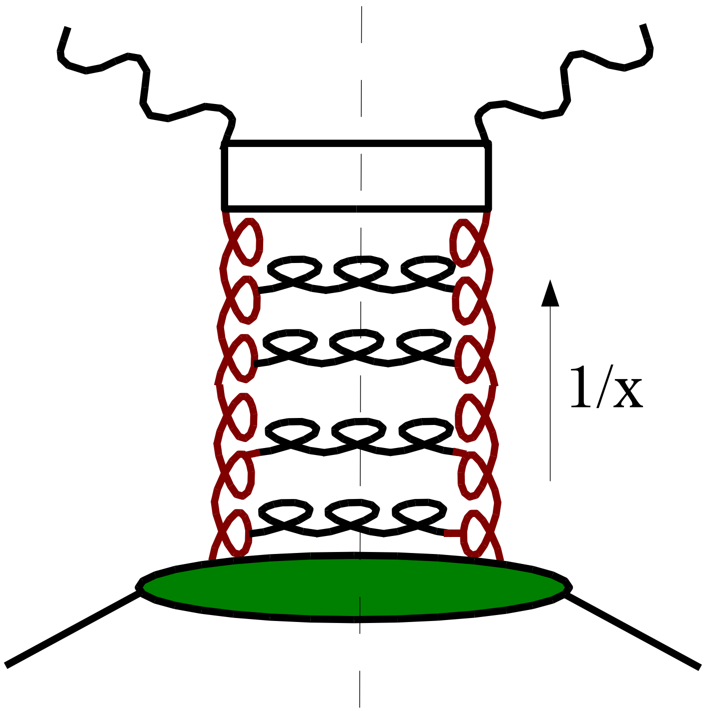


This growth can be produced as well by a different evol. eq.



[Balitsky-Fadin-Kuraev-Lipatov]

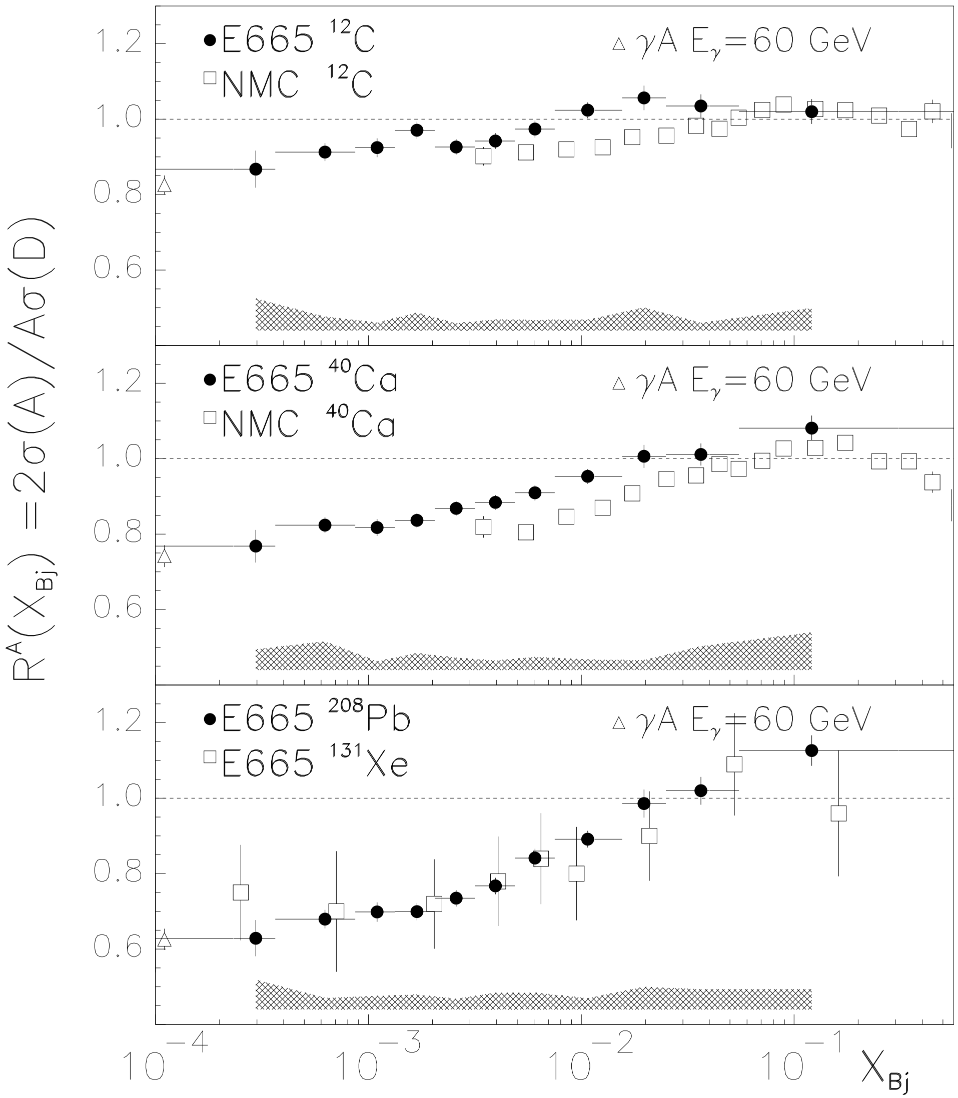
BFKL

Resums $\alpha_s \log(1/x)$

gluon distribution grows very fast at small-x

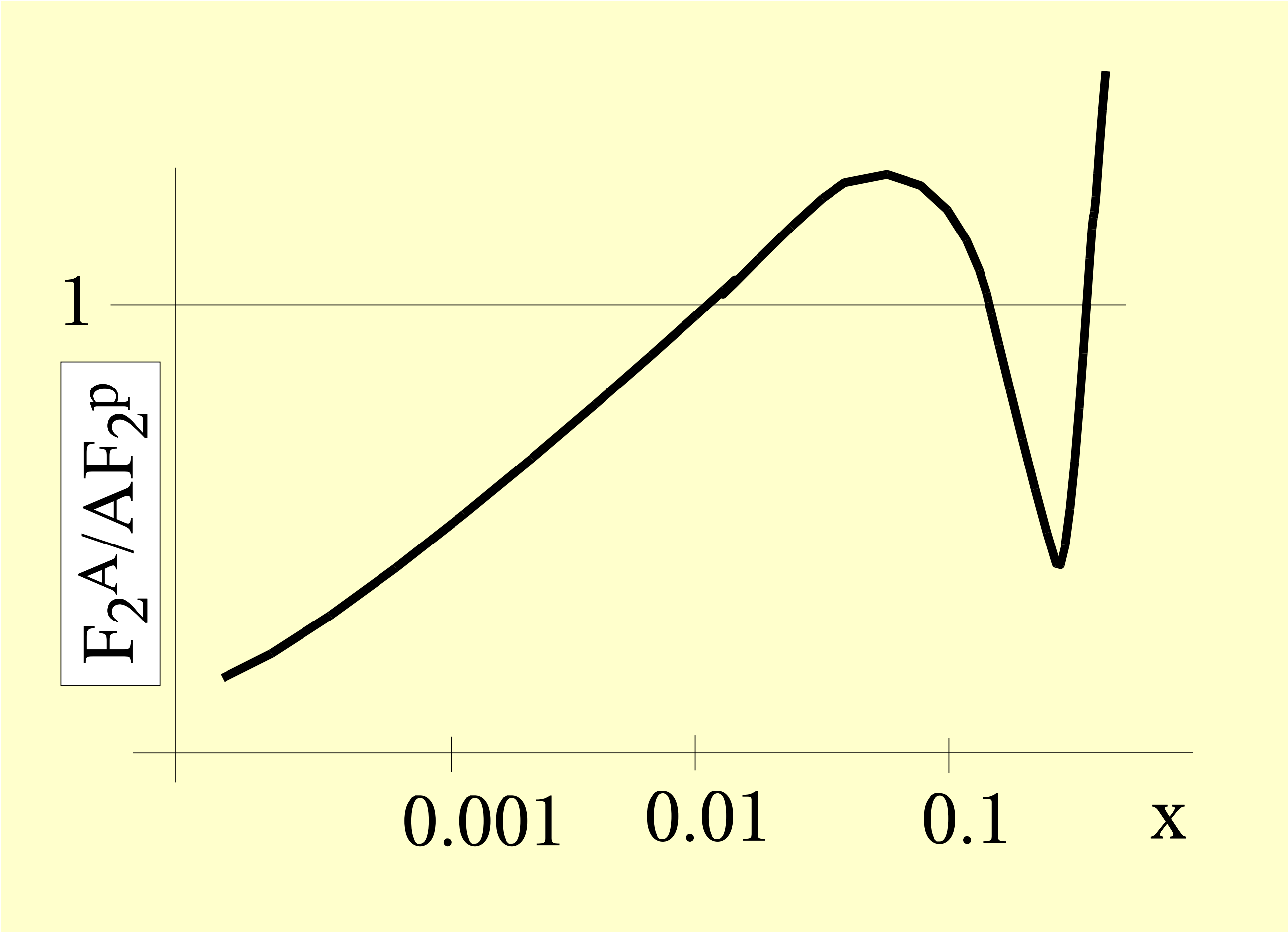
DIS experimental data with nuclei

No effect $\Rightarrow$ $F_2^A = AF_2^p$ (as the cross section is small, recall Glauber)



Nuclear structure functions

General structure: several regions



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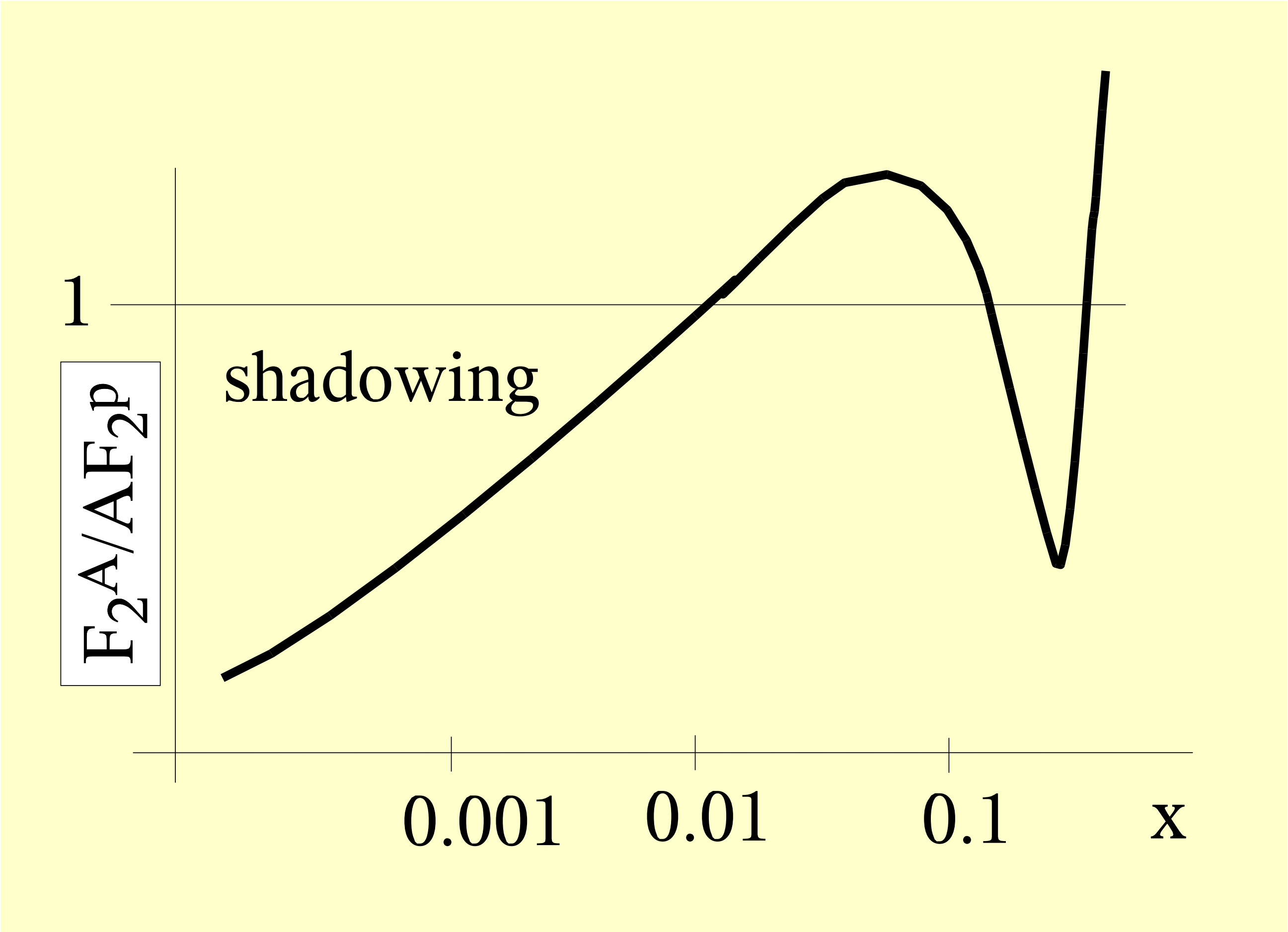

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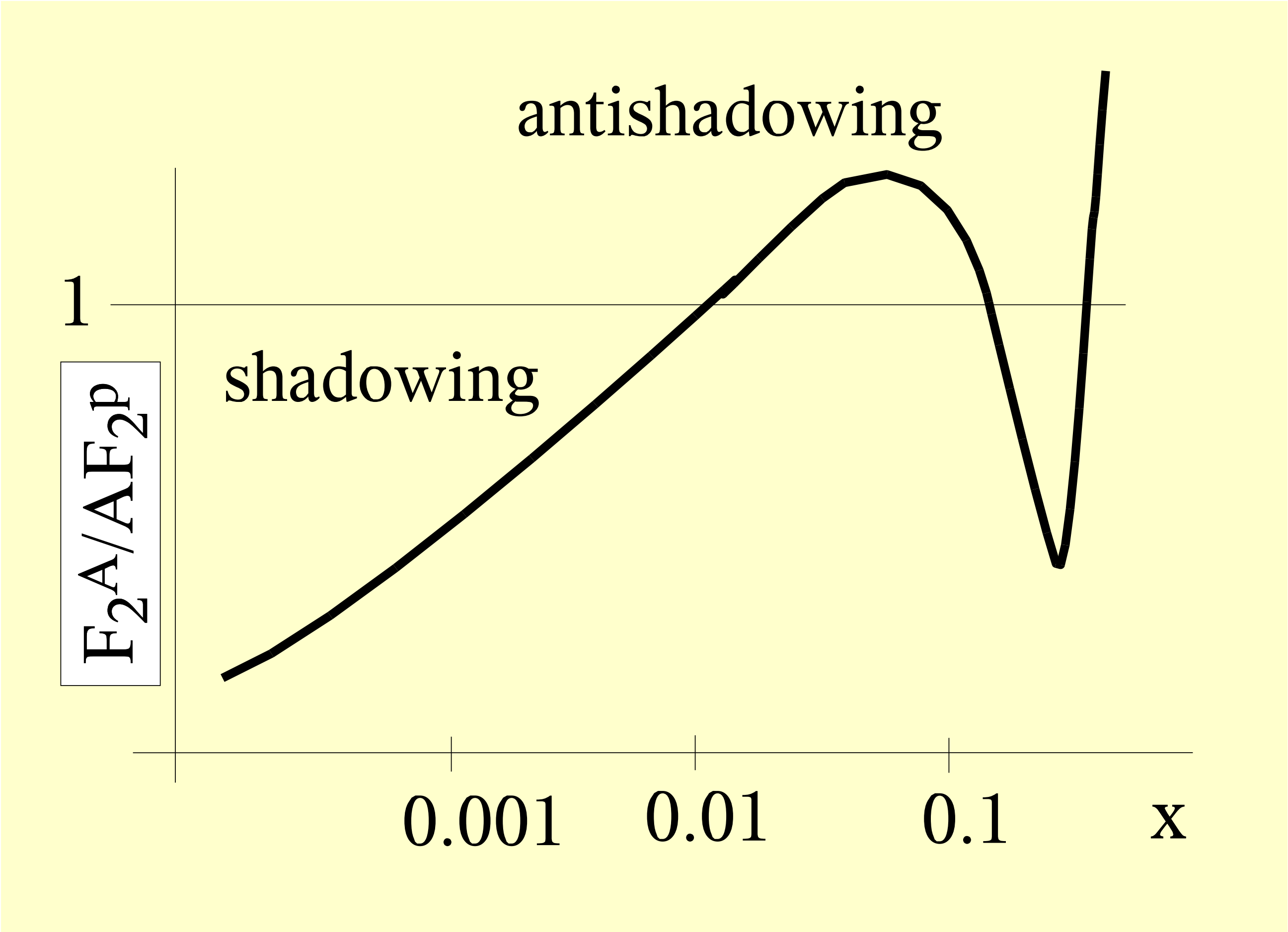

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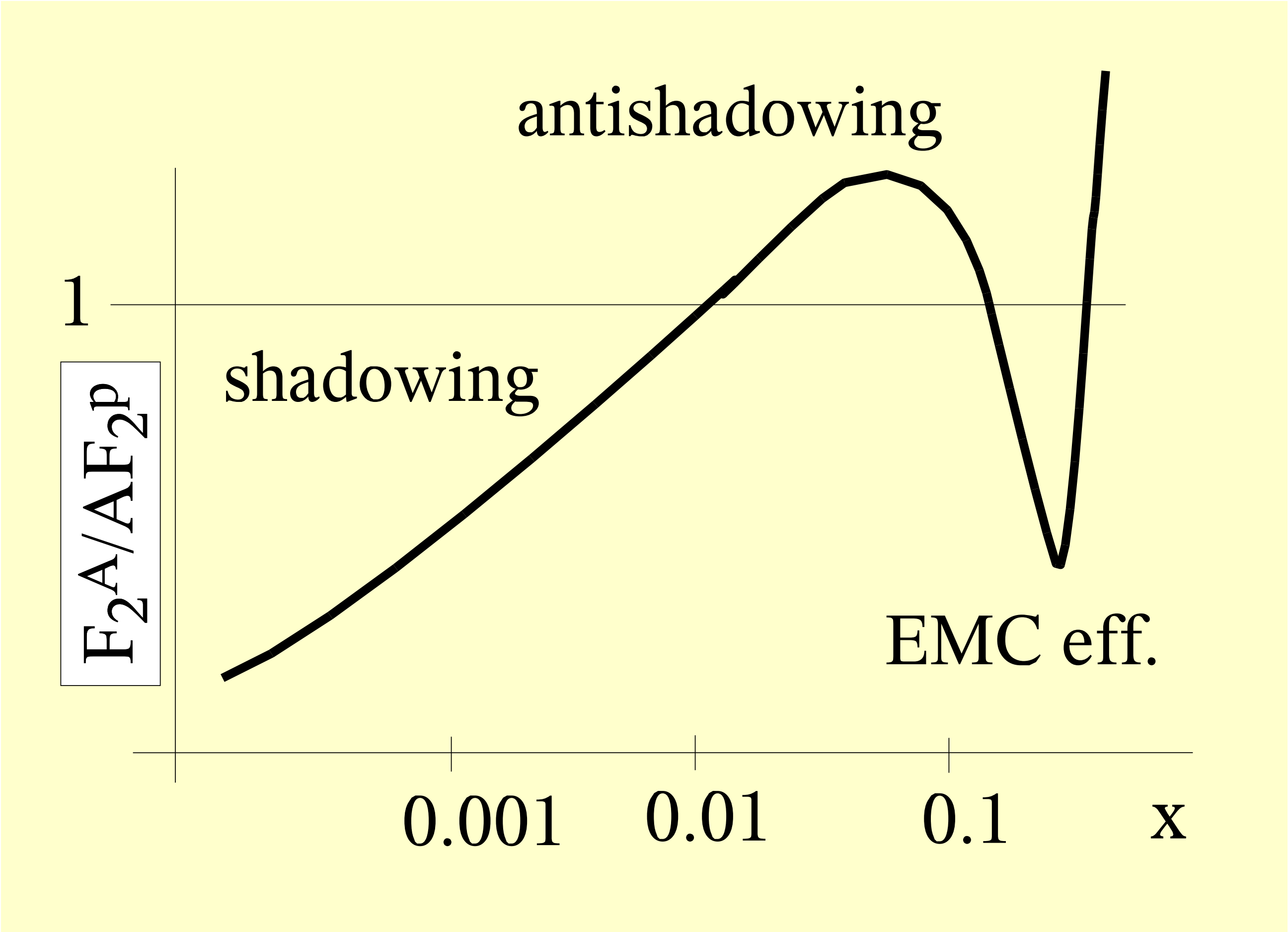

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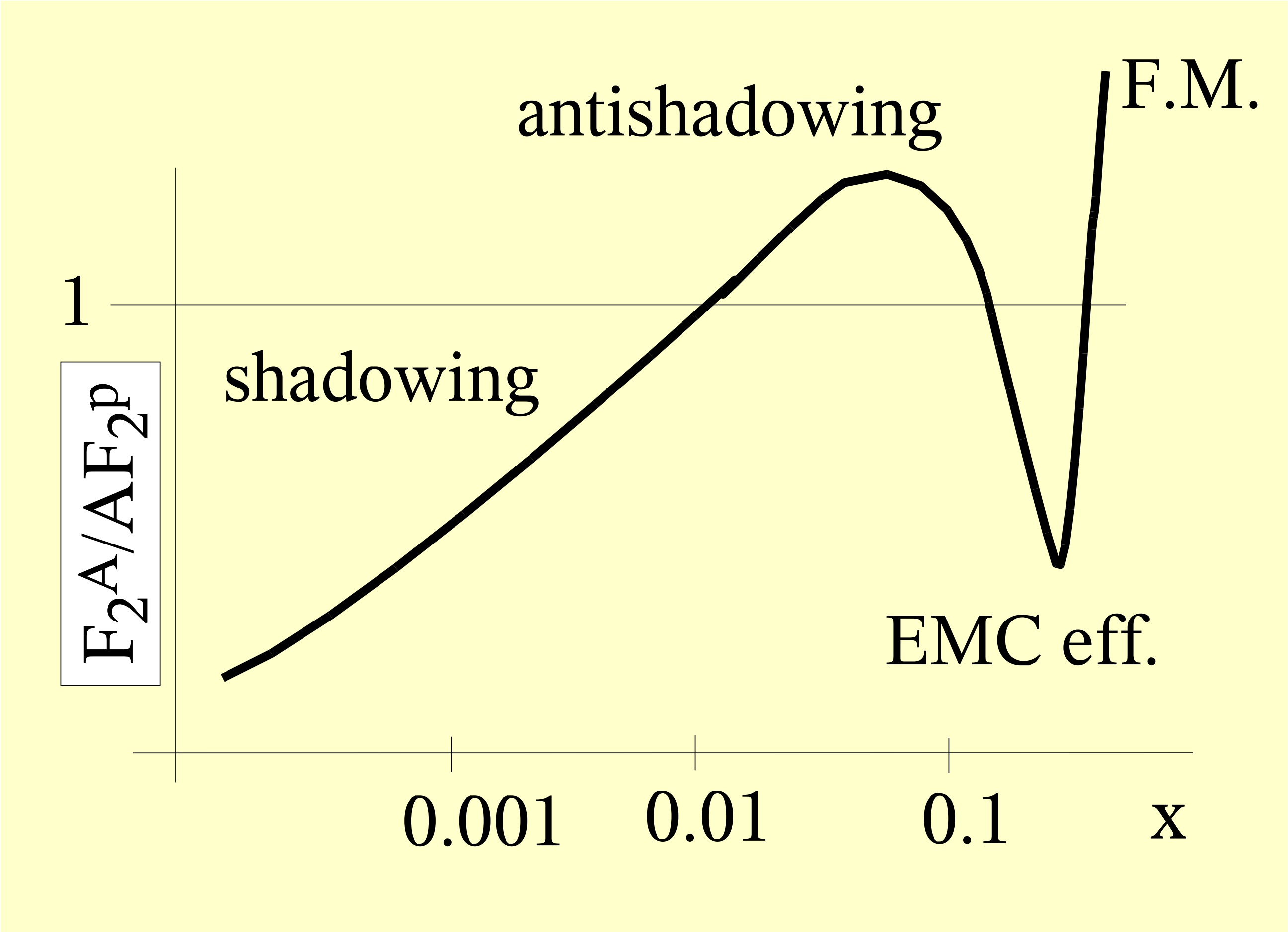
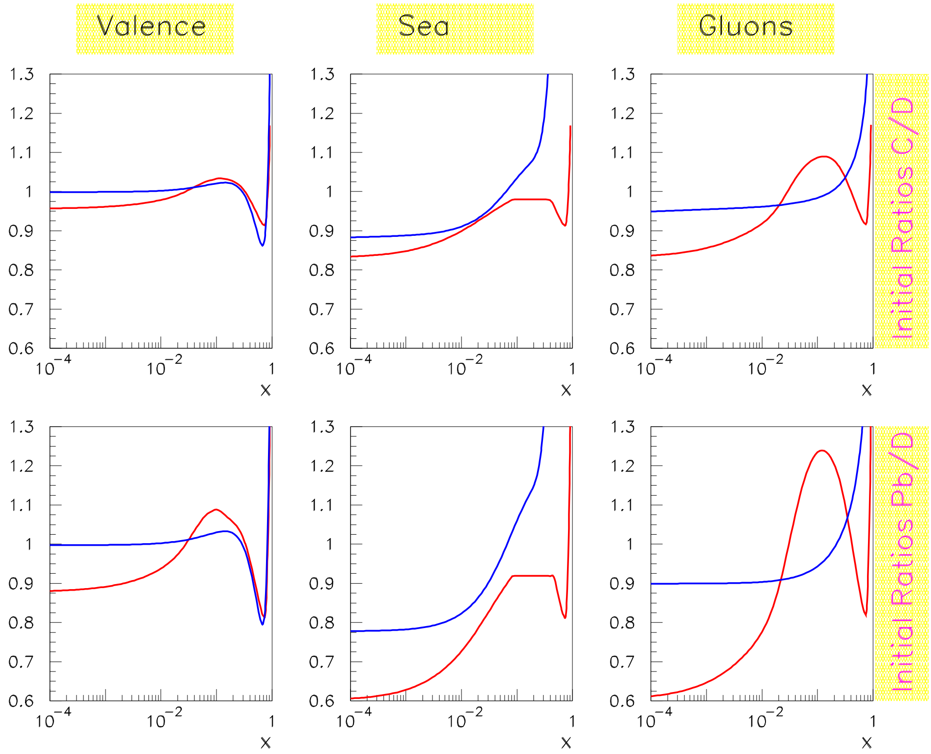


Parton distributions in the nuclei

Nuclear effects in $F_2 \longrightarrow$ nuclear effects in PDFs.

Use the method learn from proton PDF to obtain nPDF.



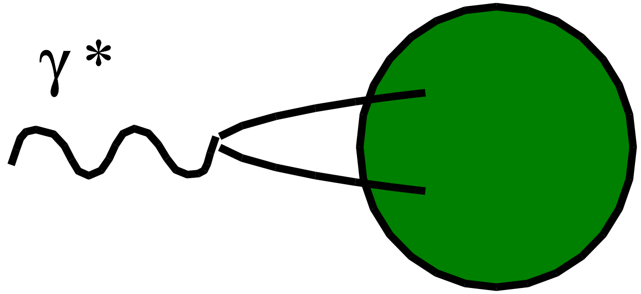
Red: Eskola, Kolhinen, Ruuskanen, Salgado

Blue: Hirai, Kumano, Miyama

Origin of shadowing

Shadowing and multiple scattering

In the laboratory frame, a $\gamma^* p$ or $\gamma^* A$ collision is a two-step process:



- ⇒ The virtual γ^* fluctuates into a $q\bar{q}$ pair
- ⇒ The $q\bar{q}$ pair interacts with de p or A

More clear space-time picture for multiple scattering

$$\sigma^{\gamma^* p(A)} = \int d^2\mathbf{r} |\Psi(\mathbf{r})|^2 \sigma^{dp(A)}(\mathbf{r}, x, Q^2)$$

where $|\Psi(\mathbf{r})|^2$ is the distribution of $q\bar{q}$ dipoles in the virtual photon and $\sigma^{dp(A)}$ is its cross section with the p or A. At leading order

$$\sigma^{dp} = \mathbf{r}^2 \frac{\pi^2}{3} \alpha_s(Q^2) x g(x, Q^2)$$

Glauber-Mueller formula for shadowing

$$\sigma^{dA} = 2\left(1 - \exp\left\{-\frac{1}{2}\sigma^{dp} A T_A(\mathbf{b})\right\}\right)$$

Saturation of partonic densities

Two properties, common to more refined calculations,are present in the Glauber-Mueller formula:

⇒ Saturation scale, defined when the term in the exponent is $\mathcal{O}(1)$

$$Q^2_{\rm sat} \sim \frac{\pi^2}{6}\alpha_s(Q^2_{\rm sat})xg(x,Q^2_{\rm sat})AT_A(\mathbf{b})$$

- $Q^2_{\rm sat} \sim x^{-\lambda}$; $\lambda \sim 0.3$ (due to the rise of $xg(x,Q^2_{\rm sat})$)
- $Q^2_{\rm sat} \sim A^{1/3}$; $AT_A(\mathbf{b}) \sim A^{1/3}$
- $Q^2_{\rm sat}$ in the perturbative region for high energy/large nuclei.

⇒ Geometric scaling: cross section only a function of $\tau = r^2 Q^2_{\rm sat} \sim Q^2/Q^2_{\rm sat}$

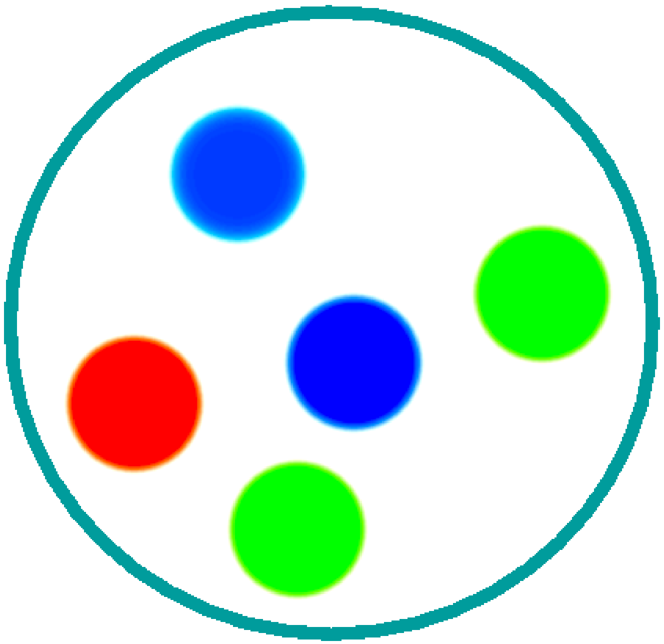
Active area of research $\longrightarrow$ Saturation Physics, Color Glass Condensate, etc...

Notice that this applies to very large energy (very small x)

$\longrightarrow$ totally coherent scattering.

Saturation of partonic densities: picture

Transverse plane, what the $\bar{q}q$ dipole sees...



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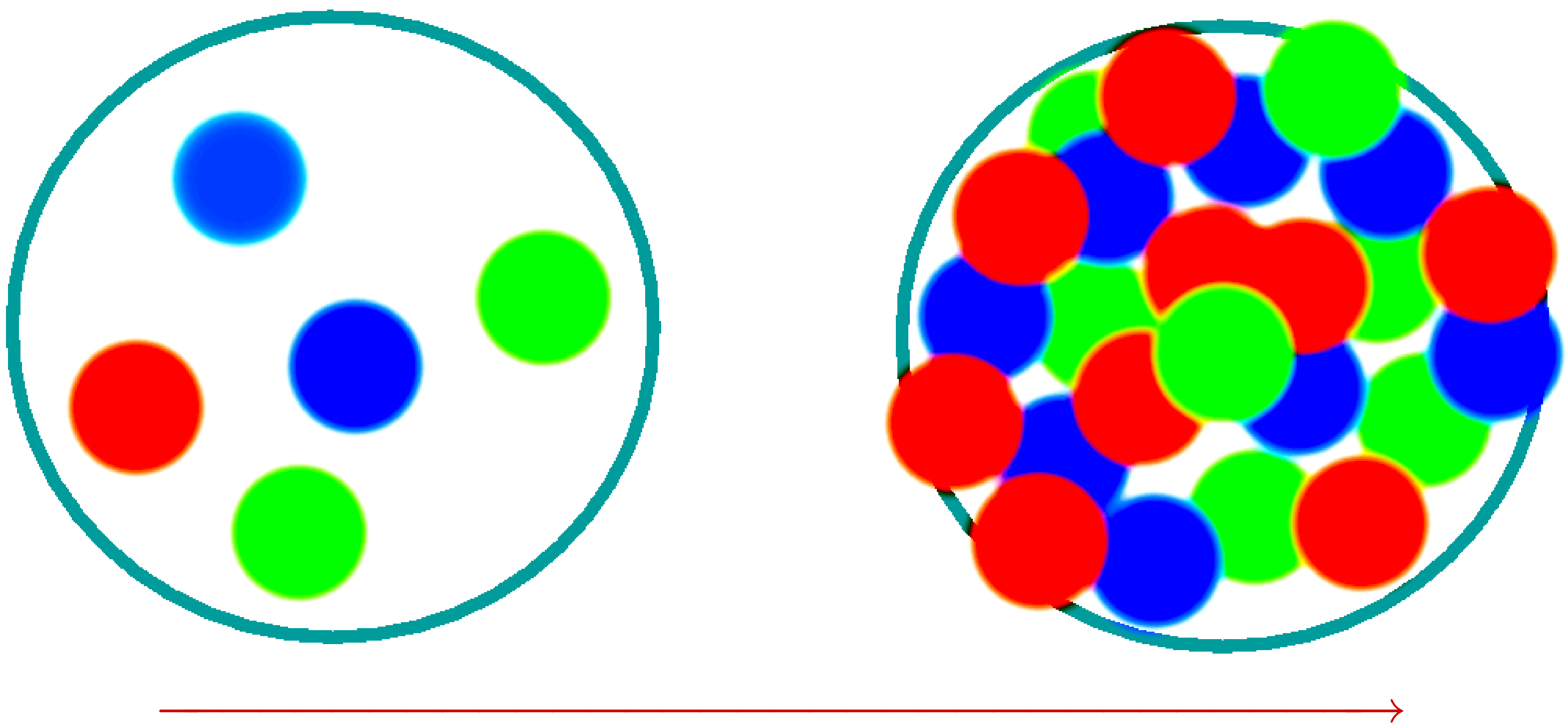


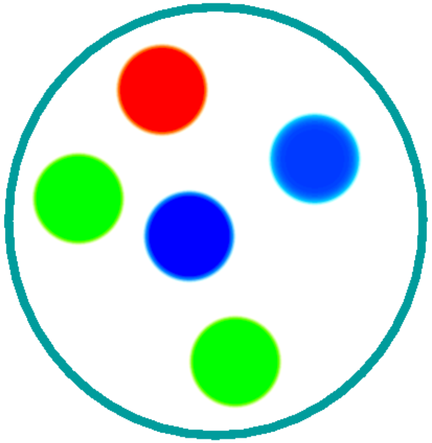
increasing energy (decreasing x)

McLerran-Venugopalan model

Distribution of gluons inside the nucleus

⇛ In the non-saturated region (large k_t)

$$\frac{dN}{dyd^2k_t} \sim \frac{1}{\alpha_s}\frac{Q^2_{\rm sat}(y)}{k_t^2}$$



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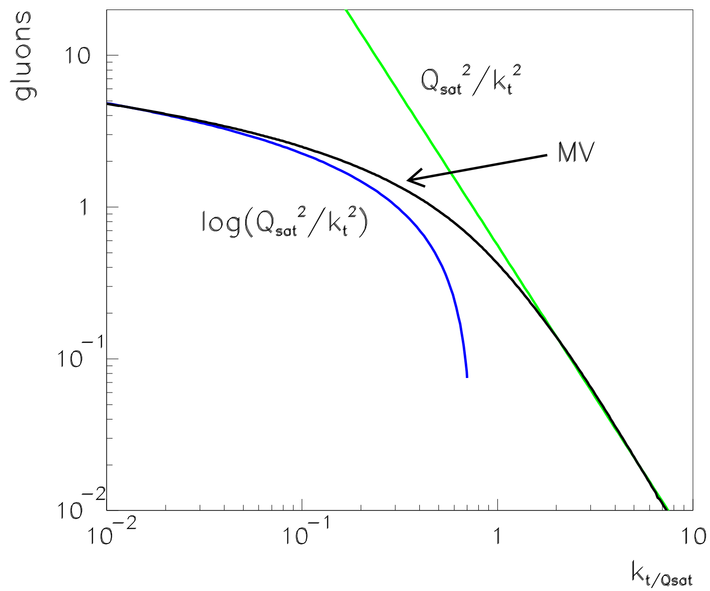
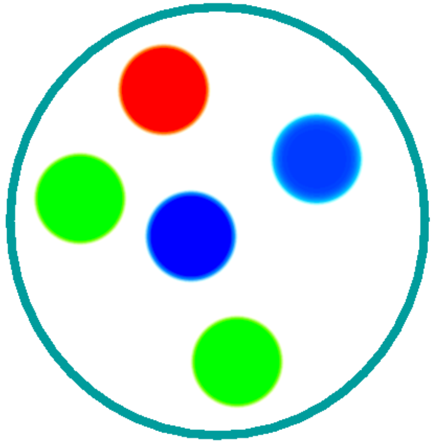
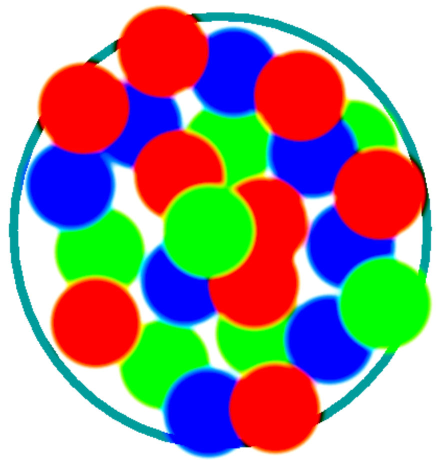
⇛ Saturation tame the growth, for small k_t

$$\frac{dN}{dyd^2k_t} \sim \frac{1}{\alpha_s} \log \frac{Q^2_{\rm sat(y)}}{k_t^2}$$

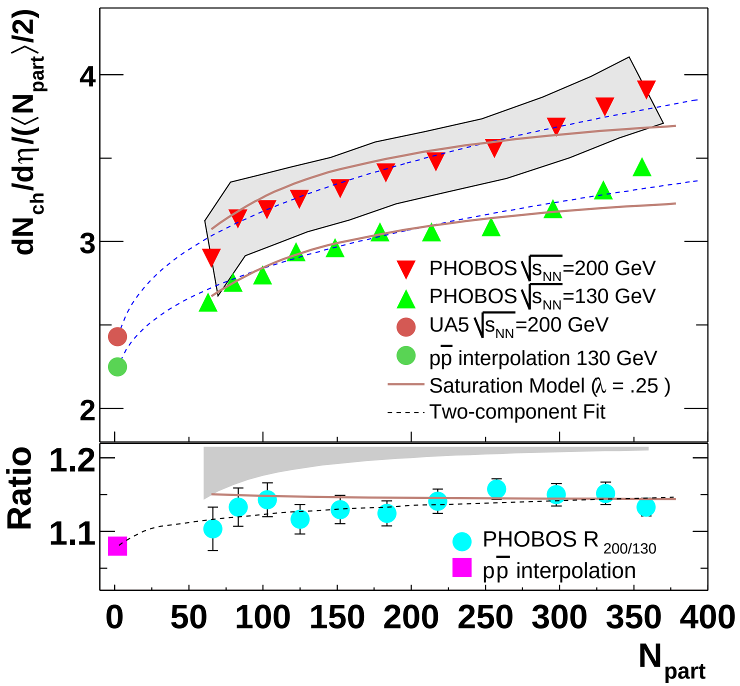


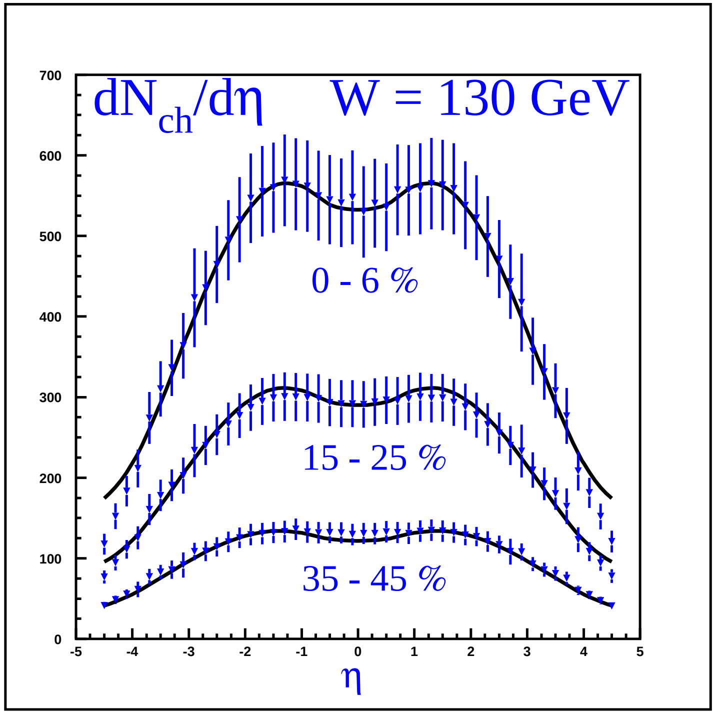




Multiplicities in nuclear collisions

- The number of particles produced in a heavy ion collision is an important quantity $\longrightarrow$ information about energy density of the medium!
- Saturation imposes a cut-off for the small-k_t gluons $\rightarrow$ the total multiplicity is given by $N \sim S_A Q^2_{\rm sat}/\alpha_s$ (where S_A is the overlap)



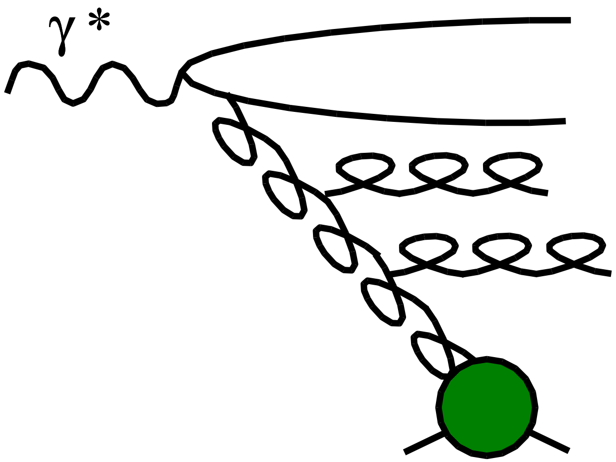


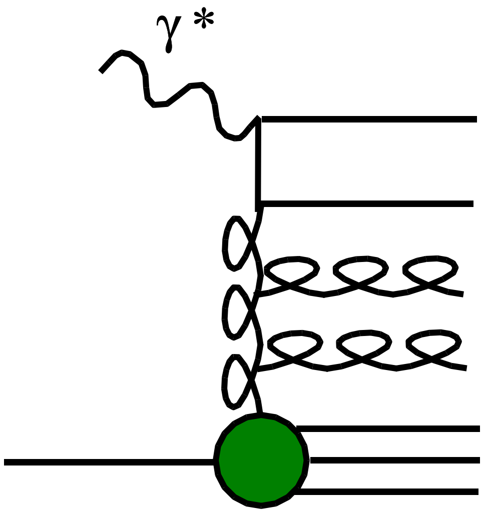
[Kharzeev *et al*]

Infinite-momentum versus laboratory frame

⇛ In the laboratory frame the proton (nucleus) is at rest

⇛ In the Bjorken frame, the proton (nucleus) has large momentum





– – – – – – – – Boost – – – – – – – – – →

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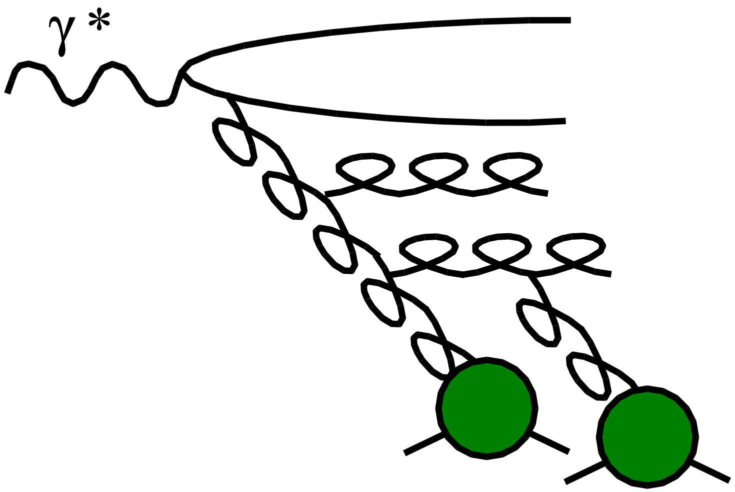

Multiple scattering

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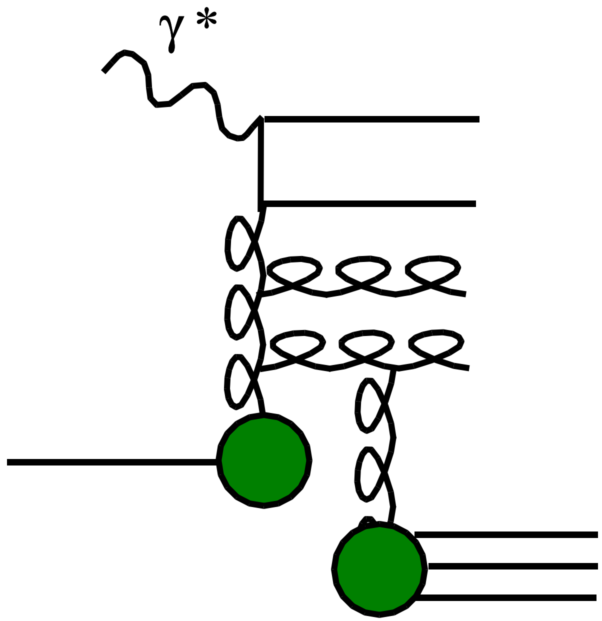


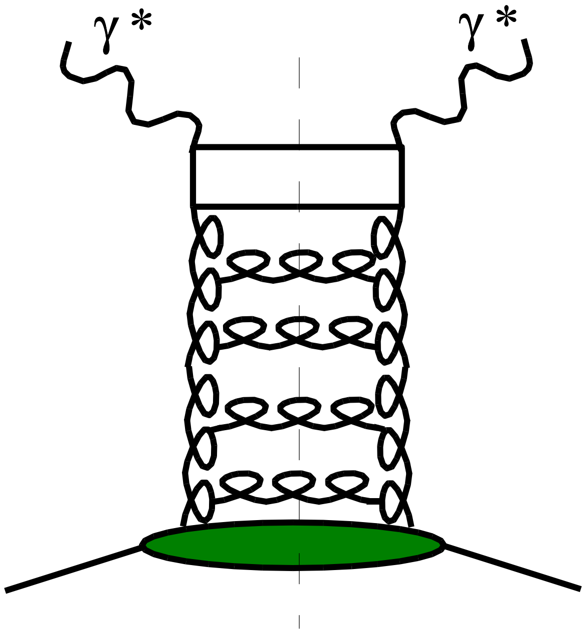
Gluon fusion

– – – – – – – – Boost – – – – – – – – – →

Multiple scattering in LAB frame $\longleftrightarrow$ gluon fusion in Bjorken frame.

Saturation and non-linear terms I

DGLAP evolution equations are linear. They take into account the splitting of a parton into two and resum logs in $1/k_\perp^2 \longrightarrow \log Q^2$.



When the density of partons (gluons) is very large (small-x) gluon fusion competes with splitting. Gribov-Levin-Ryskin-Mueller-Qiu equations

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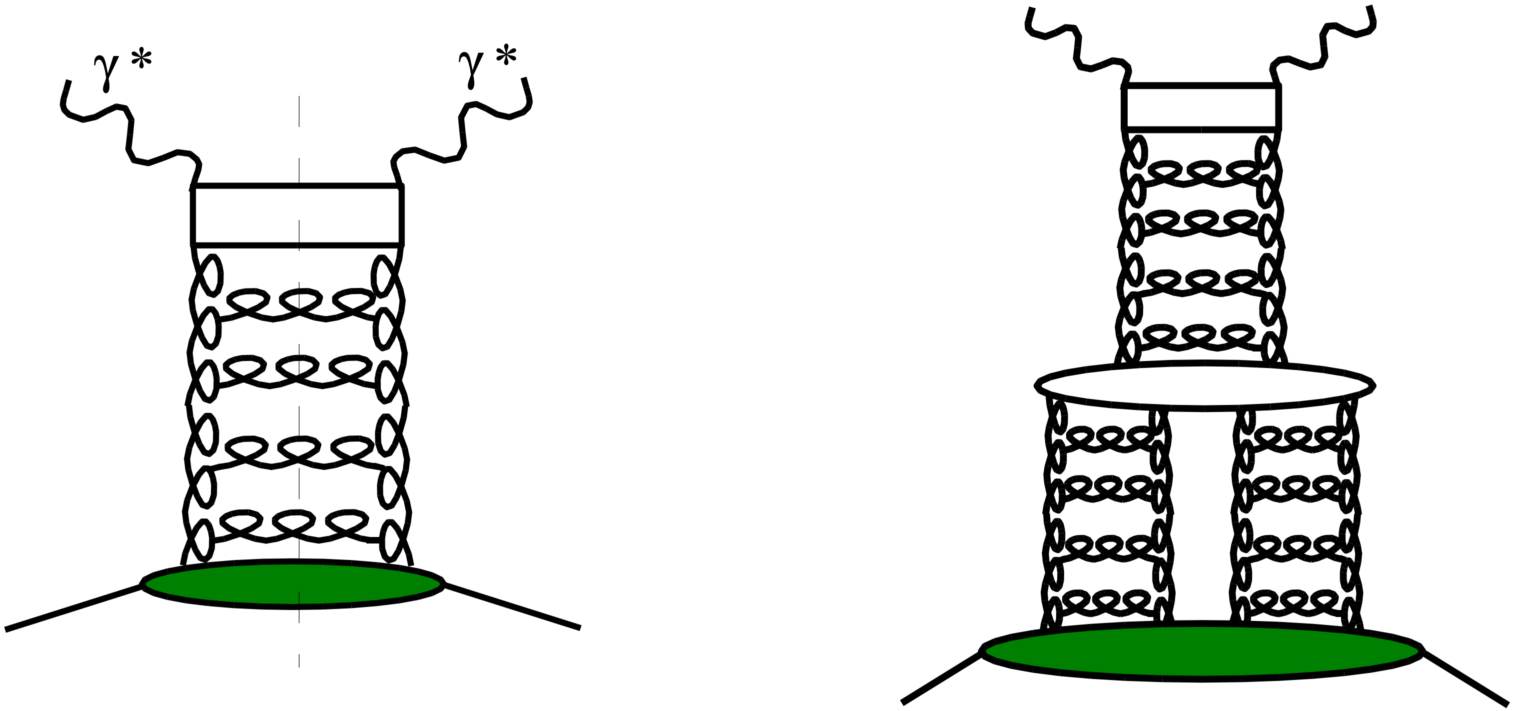

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Mueller-Qiu equations

These new terms appear as non-linear terms correcting DGLAP.

$$\frac{\partial xg(x,Q^2)}{\partial \log Q^2} = \frac{\partial xg(x,Q^2)}{\partial \log Q^2}\Big|_{DGLAP} - \frac{9}{2}\frac{\alpha_s^2}{Q^2 R^2}\int_x^1 \frac{dy}{y}\left[yg(y,Q^2)\right]^2$$

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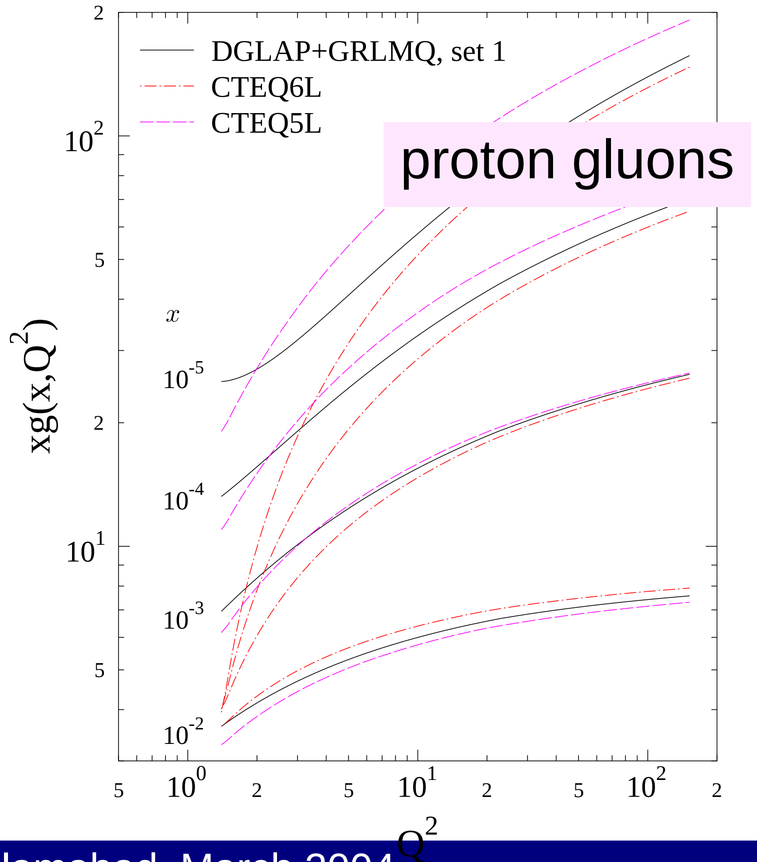

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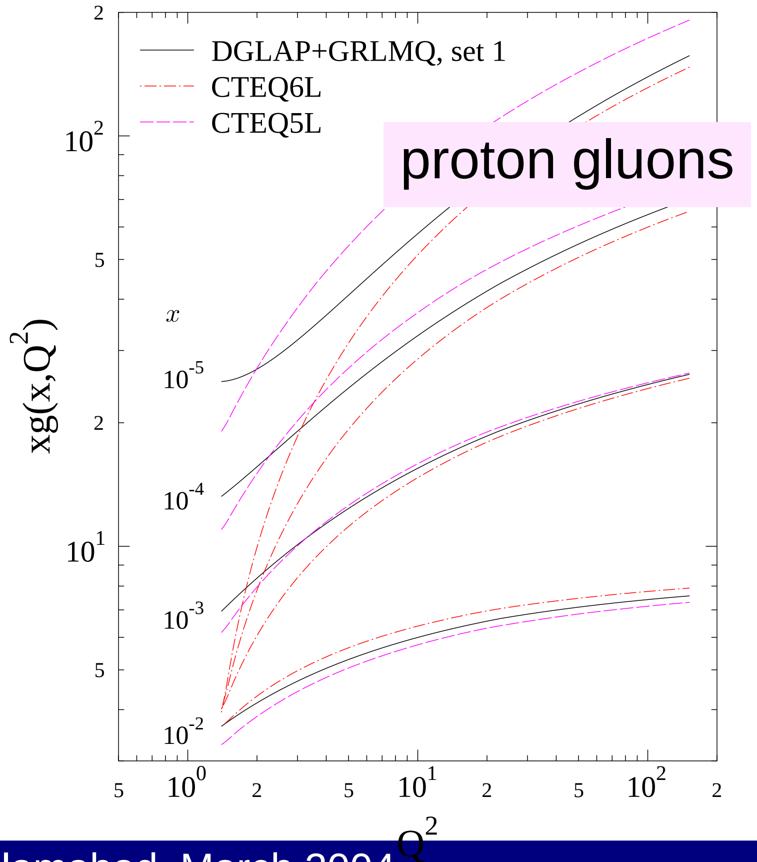

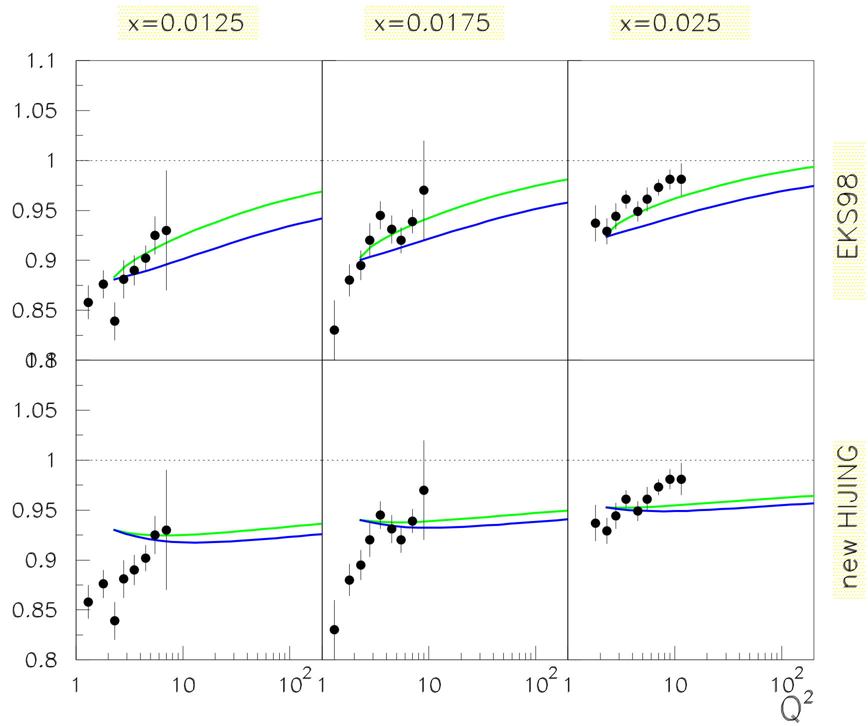

Sn/C nuclei

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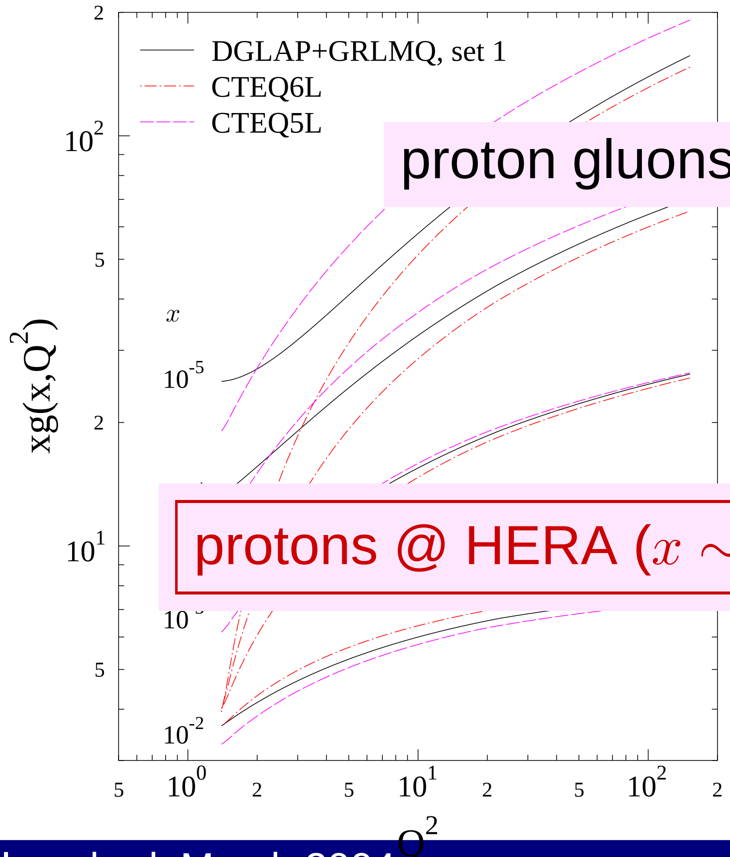
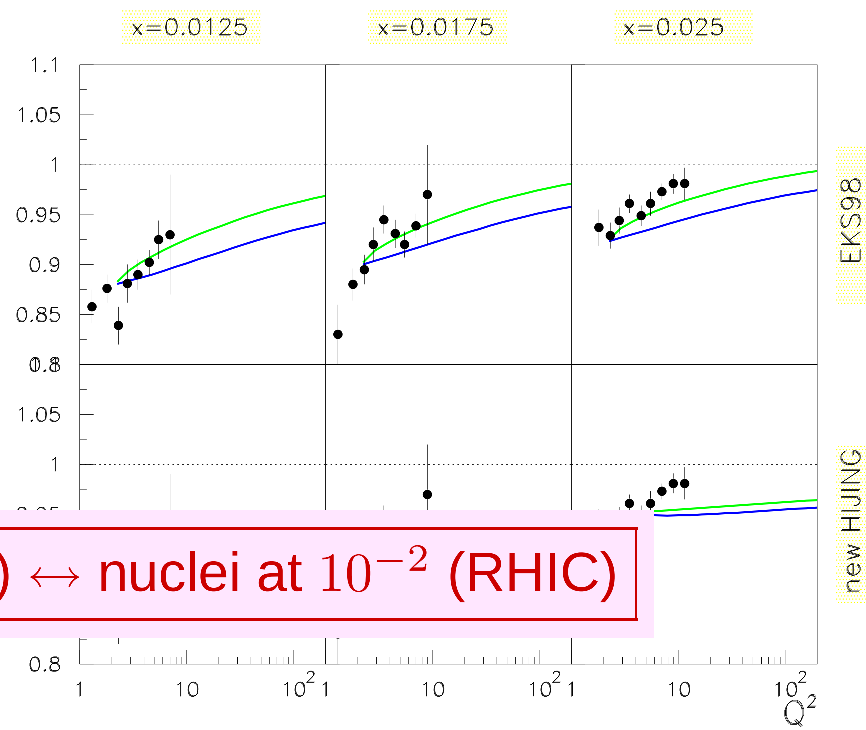


proton gluons

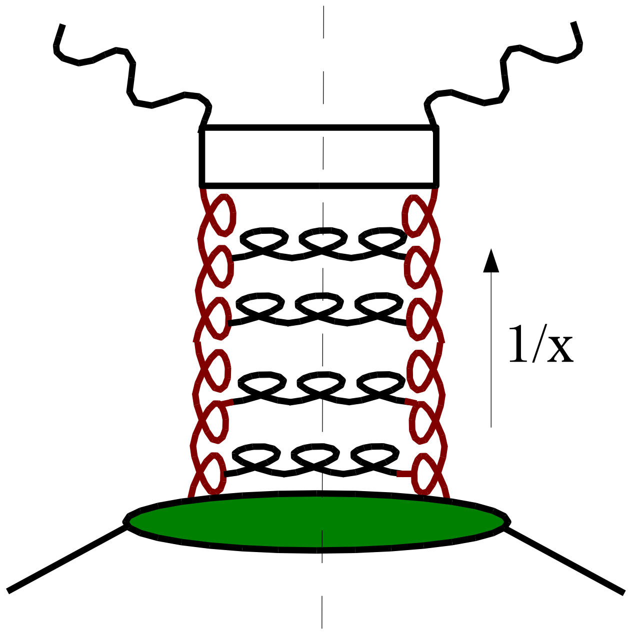


protons @ HERA ($x \sim 10^{-5}$) $\leftrightarrow$ nuclei at 10^{-2} (RHIC)

Sn/C nuclei

Saturation and non-linear terms II

In the same way, the BFKL equation needs to be corrected by nonlinear terms.



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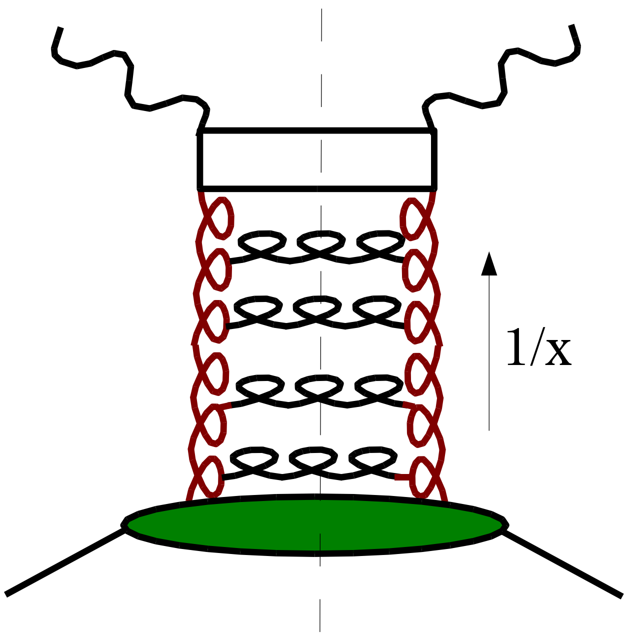

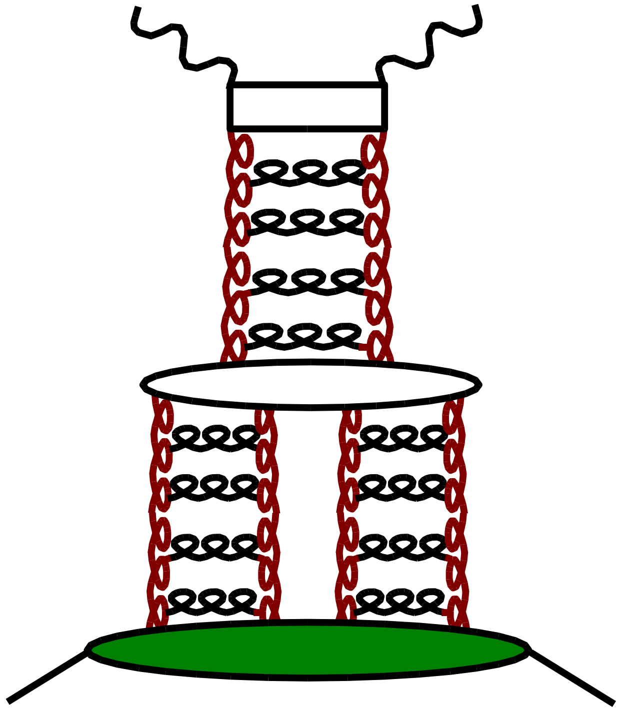

Balitsky-Kovchegov

The Balitsky-Kovchegov equation

Simple interpretation in terms of dipole amplitudes

⇒ $N(\mathbf{r}, y)$ is the forward scattering amplitude of a QCD dipole of transverse size $|\mathbf{r}|$ with rapidity Y and $y = (\alpha_s N_c/\pi) Y$,

$$\frac{dN(|\mathbf{r}|, y)}{dy} = \frac{1}{2\pi} \int d^2\mathbf{z} \, \frac{\mathbf{r}^2}{(\mathbf{r}-\mathbf{z})^2 \, \mathbf{z}^2}$$
$$\times [N(|\mathbf{r}-\mathbf{z}|) + N(|\mathbf{z}|) - N(|\mathbf{r}|) - N(|\mathbf{r}-\mathbf{z}|)N(|\mathbf{z}|)] \, .$$

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⇒ Linear terms $\longrightarrow$ BFKL equation. $N(y) \sim \mathrm{e}^{c\alpha_s y}$

- (Single) interaction of 1 dipole of size $|\mathbf{r}-\mathbf{z}|$, $|\mathbf{z}|$ and $|\mathbf{r}|$

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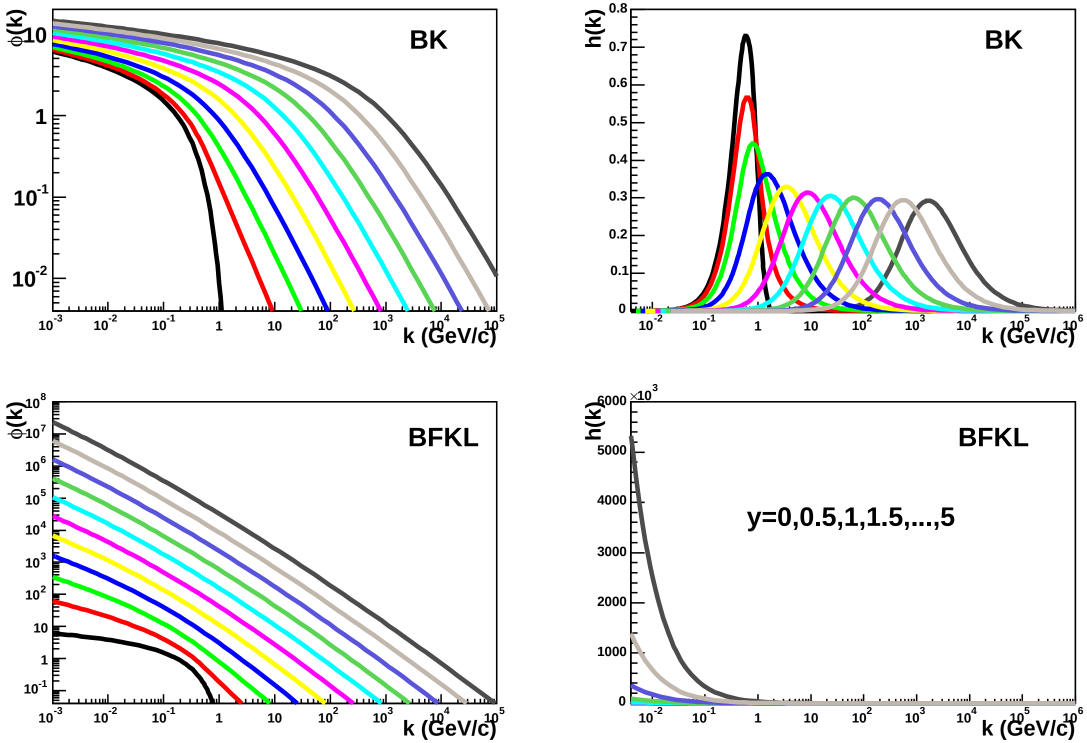
⇒ Last term is non-linear

- (Double) interaction of 2 dipoles of sizes $|\mathbf{r}-\mathbf{z}|$ and $|\mathbf{z}|$
- Makes the amplitude $N(\mathbf{r}, y)$ to be always smaller than 1. Important for the unitarity of the S-matrix.

Numerical solutions of BK

Needs initial conditions. Take McLerran-Venugopalan model (semiclassical calculations, result similar to Glauber-Mueller formula)



Albacete *et al* 2003

Two definitions for the gluon distribution:

$$\phi(k,y) = \int \frac{d^2r}{2\pi r^2} \exp\{i\mathbf{r}\cdot\mathbf{k}\} N(r,y); \qquad h(k,y) = k^2 \nabla_k^2 \phi(k,y)$$

Numerical solutions of BK

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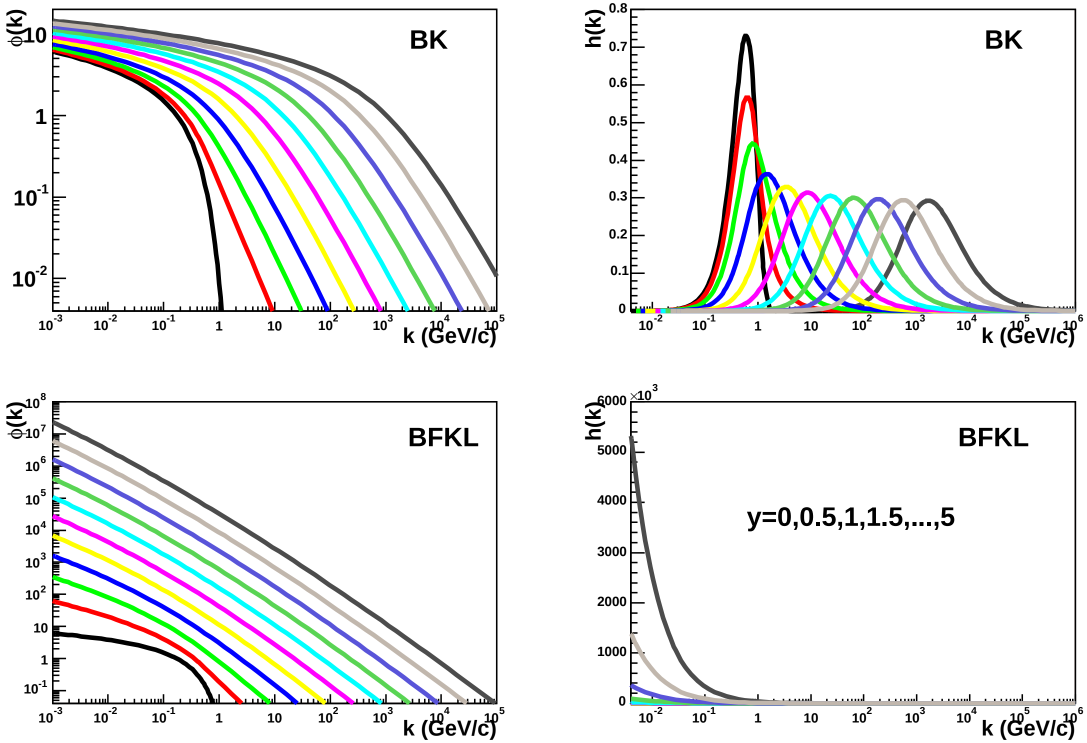
- ⇛ Non-linear terms tame the growth.
  $\rightarrow$ saturation scale.
- ⇛ Universal curve for large y: geometric scaling.



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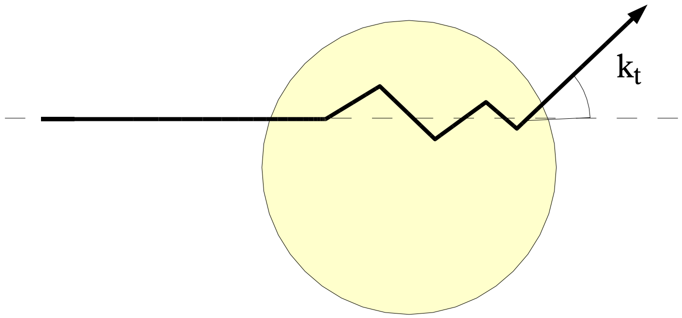
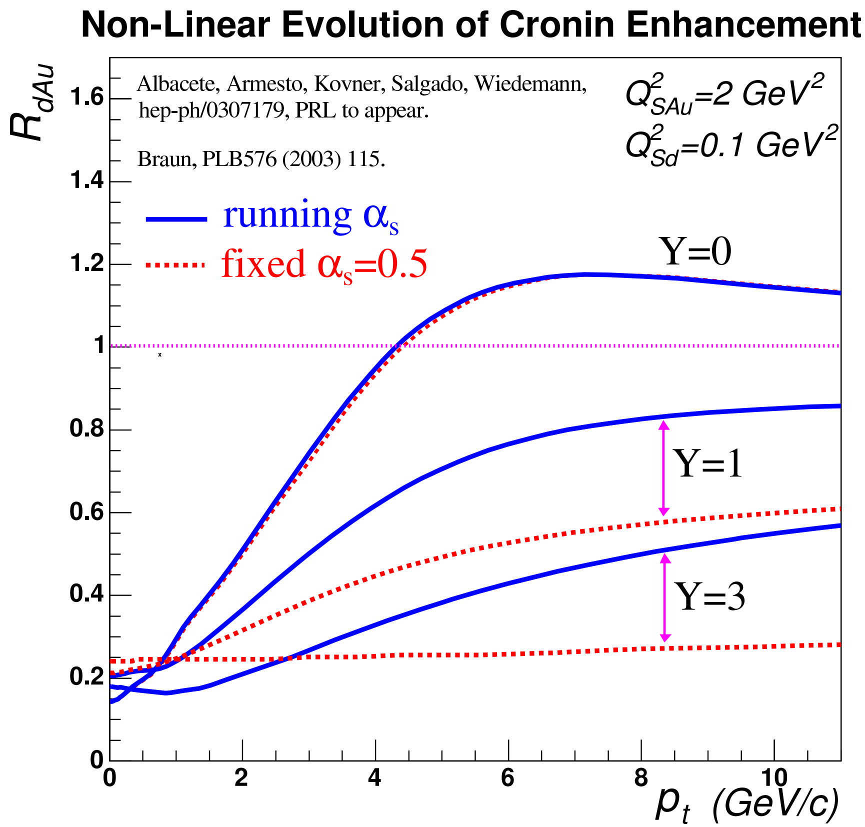
⇛ Multiple scattering produces a redistribution of the transverse momentum → Cronin effect





Ratio of gluons in a nucleus and in a proton

Cronin, anticronin...

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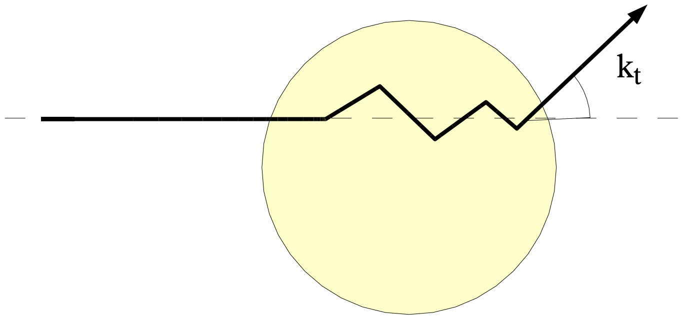
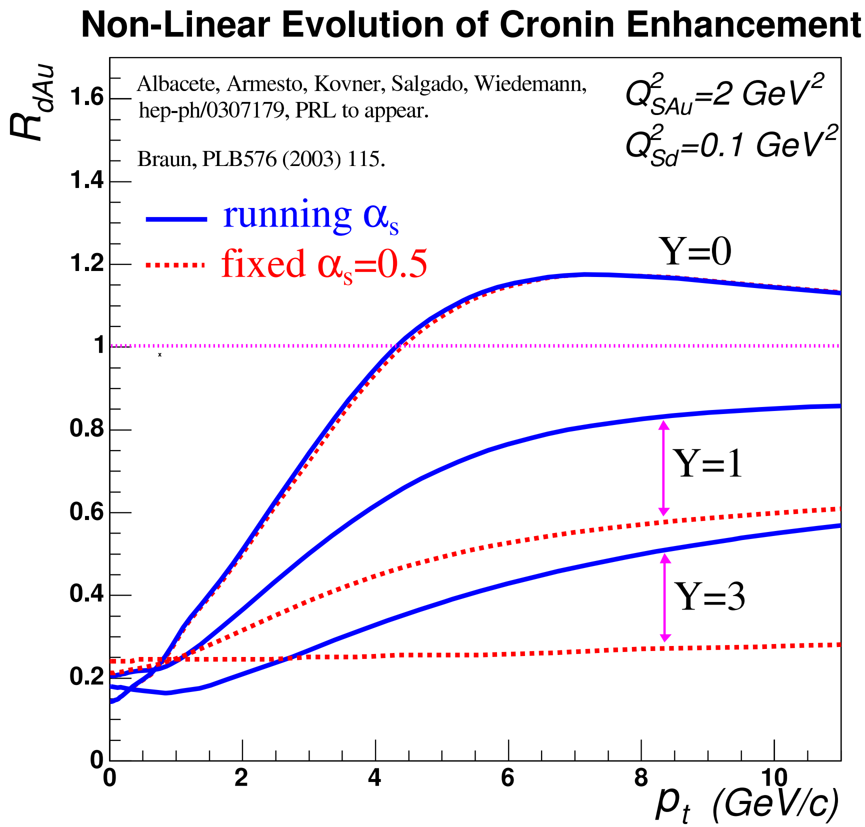


⇛ BK / BFKL Evolution removes the enhancement

⇛ Suppression of particles for all p_t at large $1/x$



Ratio of gluons in a nucleus and in a proton

Summary II

- Multiple scattering and coherence effects necessary to describe heavy ion collisions $\longrightarrow$ shadowing.
  - Nuclear parton distribution functions obtained.
- Totally coherent multiple scattering $\longrightarrow$ saturation.
- Non-linear terms in the evolution equations appear.
- Observable consequences of saturation
  - Multiplicities in heavy ion collisions
  - Suppression of high-p_t particles
- Initial conditions for heavy ion collisions.