

# CP-Violation and Rare *B*-Decays in the SM & SUSY

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4th. Particle Physics Workshop, NCP, Islamabad

## *Plan of Talk*

- Flavour Mixing in the SM & SUSY
- CP-Violation in  $B$ -decays & possible SUSY effects
- Rare  $B$ -decays in the SM & SUSY
- Summary

## The Cabibbo-Kobayashi-Maskawa Matrix

$$V_{\text{CKM}} \equiv \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

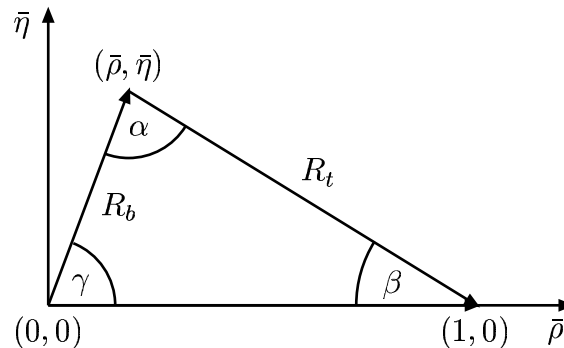
- Customary to use the handy **Wolfenstein parametrization**

$$V_{\text{CKM}} \simeq \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda(1 + iA^2\lambda^4\eta) & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2(1 + i\lambda^2\eta) & 1 \end{pmatrix}$$

- Four parameters:  $A$ ,  $\lambda$ ,  $\rho$ ,  $\eta$
- Perturbatively improved version of this parametrization

$$\bar{\rho} = \rho(1 - \lambda^2/2), \quad \bar{\eta} = \eta(1 - \lambda^2/2)$$

- The CKM-Unitarity triangle [ $\phi_1 = \beta$ ;  $\phi_2 = \alpha$ ;  $\phi_3 = \gamma$ ]



## Superfield Classification in the MSSM

- Superfields classified according to their  $SU(3)_C \otimes SU(2)_I \otimes U(1)_Y$  Quantum Numbers;  $i = 1, 2, 3$  a generation index

- Chiral Superfields for Quarks ( $\hat{Q}_i, \hat{U}_i^c, \hat{D}_i^c$ )

$$\hat{Q}_i(3, 2, 1/6); \quad \hat{U}_i^c(\bar{3}, 1, -2/3); \quad \hat{D}_i^c(\bar{3}, 1, 1/3)$$

$$\hat{Q}_i = (\tilde{Q}_{L_i}, Q_{L_i}); \quad \hat{U}_i^c = (\tilde{U}_{L_i}^c, U_{L_i}^c); \quad \hat{D}_i^c = (\tilde{D}_{L_i}^c, D_{L_i}^c)$$

- Chiral Superfields for Leptons ( $\hat{L}_i, \hat{E}_i^c$ )

$$\hat{L}_i(1, 2, -1/2); \quad \hat{E}_i^c(1, 1, 1)$$

$$\hat{L}_i = (\tilde{E}_{L_i}, E_{L_i}); \quad \hat{E}_i^c = (\tilde{E}_{L_i}^c, E_{L_i}^c)$$

- Chiral Superfields for Two Higgs Doublets (also denoted as  $\hat{H}_1$  &  $\hat{H}_2$ )

$$\hat{H}_u(1, 2, -1/2); \quad \hat{H}_d(1, 2, 1/2)$$

$$\hat{H}_u = (H_u, \tilde{H}_u); \quad \hat{H}_d = (H_d, \tilde{H}_d)$$

- Vector Superfields ( $\hat{G}, \hat{W}, \hat{B}$ ) ( $\alpha$  is an  $SU(2)$  index)

$$\hat{G}(8, 1, 1); \quad \hat{W}^\alpha(1, 3, 1); \quad \hat{B}(1, 1, 1)$$

$$\hat{G} = (g, \tilde{g}); \quad \hat{W} = (W^\alpha, \tilde{W}^\alpha); \quad \hat{B} = (B, \tilde{B})$$

## Flavour Mixing in the MSSM

- Flavour mixings in the MSSM reside in the Superpotential  $W_{\text{MSSM}}$  and in the soft supersymmetry-breaking Lagrangian  $\mathcal{L}_{\text{soft}}$
- Assume  $R$ -parity:  $R = (-1)^{3(B-L)+2S}$ ; where  $B(L)$  is Baryon (Lepton) quantum number and  $S$  is the spin: SM particles are  $R$ -even, supersymmetric particles are  $R$ -odd
- $W_{\text{MSSM}}$

$$W_{\text{MSSM}} = \epsilon_{\alpha\beta} [-\hat{H}_u^\alpha \hat{Q}_i^\beta Y_u^{ij} \hat{U}_j^c + \hat{H}_d^\alpha \hat{Q}_i^\beta Y_d^{ij} \hat{D}_j^c + \hat{H}_d^\alpha \hat{L}_i^\beta Y_e^{ij} \hat{E}_j^c - \mu \hat{H}_d^\alpha \hat{H}_u^\beta]$$

$$\epsilon_{\alpha\beta} = -\epsilon_{\beta\alpha}; \quad \epsilon_{12} = 1$$

- $\mathcal{L}_{\text{soft}}$

$$\mathcal{L}_{\text{soft}} = \frac{1}{2} [M_3 \tilde{g} \tilde{g} + M_2 \tilde{W}^\alpha \tilde{W}^\alpha + M_1 \tilde{B} \tilde{B} + h.c.]$$

$$+ \epsilon_{\alpha\beta} [-b H_d^\alpha H_u^\beta - H_u^\alpha \tilde{Q}_i^\beta \tilde{A}_{u_{ij}} \tilde{U}_j^c + H_d^\alpha \tilde{Q}_i^\beta \tilde{A}_{d_{ij}} \tilde{D}_j^c + H_d^\alpha \tilde{L}_i^\beta \tilde{A}_{e_{ij}} \tilde{E}_j^c + h.c.]$$

$$+ m_{H_d}^2 |H_d|^2 + m_{H_u}^2 |H_u|^2 + \tilde{Q}_i^\alpha m_{Q_{ij}}^2 \tilde{Q}_j^{\alpha*}$$

$$+ \tilde{L}_i^\alpha m_{L_{ij}}^2 \tilde{L}_j^{\alpha*} + \tilde{U}_i^{c*} m_{U_{ij}}^2 \tilde{U}_j^c + \tilde{D}_i^{c*} m_{D_{ij}}^2 \tilde{D}_j^c + \tilde{E}_i^{c*} m_{E_{ij}}^2 \tilde{E}_j^c$$

- MSSM contains 124 parameters residing in the Superpotential  $W_{\text{MSSM}}$  (Yukawa couplings) and Soft-SUSY-breaking  $\mathcal{L}_{\text{soft}}$  (Scalar) terms
- Various realizations of the MSSM differ from each other in the details of  $\mathcal{L}_{\text{soft}}$

## SUGRA and mSUGRA models

- CKM matrix is the only source of Flavour transitions
- In SUGRA models, this is achieved by assuming that the SUSY-breaking parameters have a simple structure at the GUT scale ( $m_X$ )

$$(m_Q^2)_j^i = (m_E^2)_j^i = (m_D^2)_j^i = (m_U^2)_j^i = (m_L^2)_j^i = M_0^2 \delta_j^i$$

$$m_{H_d}^2 = m_{H_u}^2 = \Delta_0^2$$

$$M_1 = M_2 = M_3 = M_{1/2}$$

$$A_{d_{ij}} = A_0 (Y_d)_{ij}; \quad A_{u_{ij}} = A_0 (Y_u)_{ij}; \quad A_{e_{ij}} = A_0 (Y_e)_{ij}$$

- In MSUGRA model, in addition  $\Delta_0^2 = M_0^2$
- RG running ( $m_X \rightarrow m_W$ ) induces flavour non-diagonal terms, but they are small
- This reduces the number of parameters enormously, leaving the parameters:  $M_0$ ,  $M_{1/2}$ ,  $|A_0|$ ,  $\tan \beta$ ,  $\phi_\mu$ ,  $\phi_A$ , where the phases are constrained by the EDMs
- Minimal flavour violation (MFV) models are highly predictive, and hence highly constrained

## General Flavour Violating SUSY & The MIA Technique

- In a general SUSY Model, many more sources of Flavour Violation
- A technique to carry out an analysis in a general SUSY framework is the Mass Insertion Approximation (MIA) [Hall, Kostelecky, Raby 1986]
- In the MIA approach, one choses a basis in which the couplings of  $\tilde{f}_i \tilde{g} f_j$  are flavour-diagonal ( $\propto \delta_{ij}$ ); FC take place on the sfermion propagators by mass insertions:  $\Delta_{ij}^u, \Delta_{ij}^d$  etc.

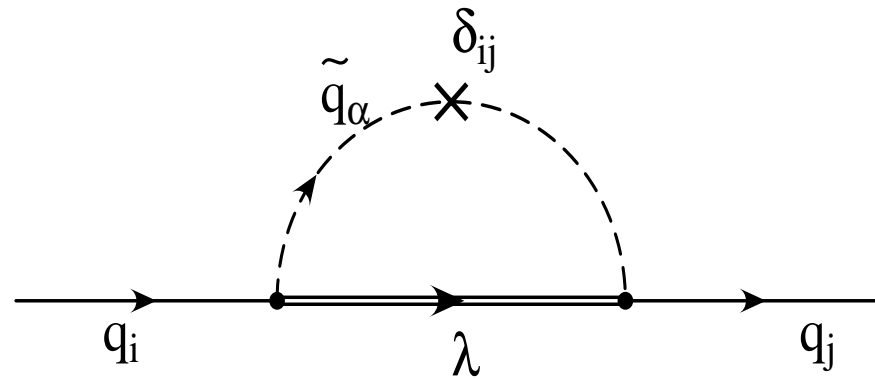
$$(m_0^2)_{ij} \delta_{ij} + \Delta_{ij}$$

- Need not know the full diagonalization of the sfermion ( $\tilde{f}$ ) mass matrices; sufficient to compute the ratios ( $\langle m_0^2 \rangle$  is an average sfermion mass squared):

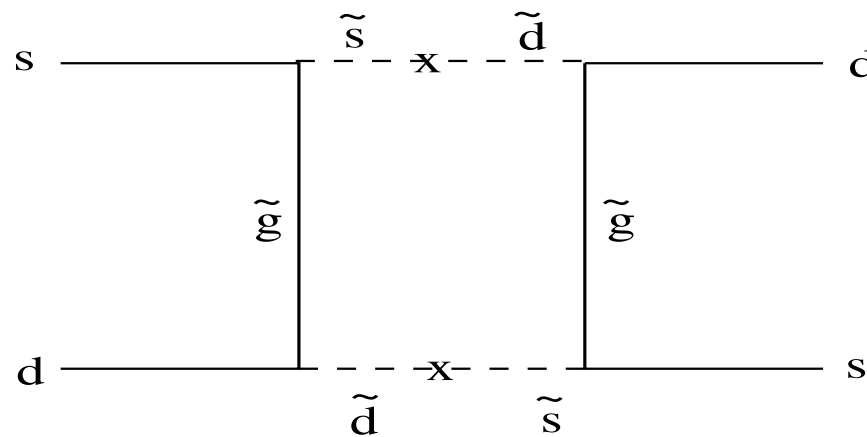
$$\delta_{ij} = \frac{\Delta_{ij}}{\langle m_0^2 \rangle}$$

- All FC effects can be parametrized in terms of a limited number of complex MIA parameters:  $(\delta_{ij}^u)_{AB}$  &  $(\delta_{ij}^d)_{AB}$ , ( $A, B = L, R$ )
- Typically, one expects  $(\delta_{ij}^f)_{AB} \leq 1$
- Analysis for FV processes can then be carried out in terms of the SUSY-MFV contributions and the MIA parameters [Masiero et al.,...]

## Typical Mass Insertion Diagrams



A SUSY contribution for  $K^0 - \bar{K}^0$  Mixing

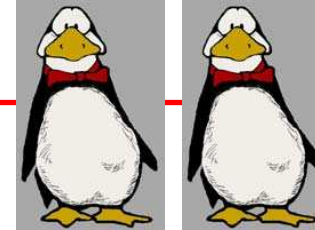


- For  $B_d^0 - \bar{B}_d^0$  Mixings:  $s \rightarrow b$ ;  $\tilde{s} \rightarrow \tilde{b}$
- For  $B_s^0 - \bar{B}_s^0$  Mixings:  $d \rightarrow b$ ;  $\tilde{d} \rightarrow \tilde{b}$

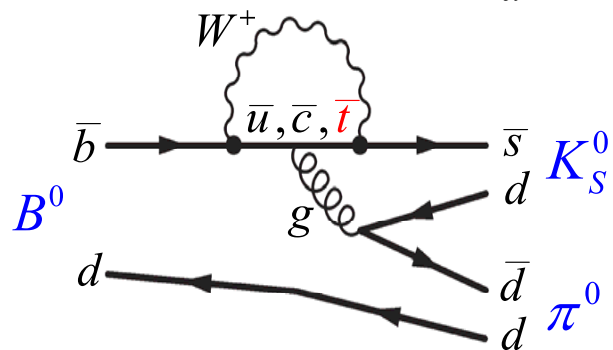
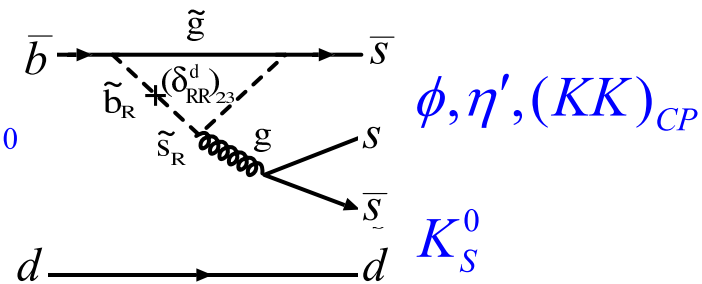
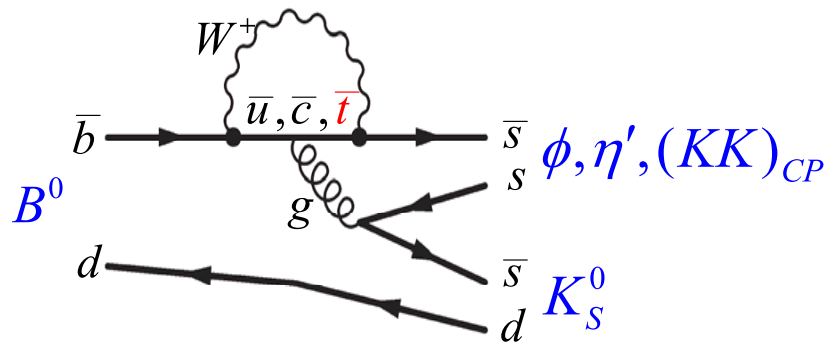
## A list of flavour-violating observables

Observable	SM Prediction	MSSM-MIA (AB=LL, LR, RL, RR)
$\Delta m_K$	$\sim (V_{cs}^* V_{cd})^2$	$(\delta_{AB})_{12}$
$\epsilon$	$\sim \text{Im}(V_{ts}^* V_{td}) \text{Re}(V_{cs}^* V_{cd})$	$(\delta_{AB})_{12}$
$\epsilon'/\epsilon$	$\sim \text{Im}(V_{ts}^* V_{td})$	$(\delta_{AB})_{12}$
$b \rightarrow s\gamma$	$\sim V_{tb} V_{ts}^*$	$(\delta_{AB})_{23}$
$A_{CP}(b \rightarrow s\gamma)$	$\sim \alpha_s(m_b) \frac{V_{ub}}{V_{cb}} \frac{m_c^2}{m_b^2}$	$(\delta_{AB})_{23}$
$\Delta m_{B_d}$	$\sim (V_{td}^* V_{tb})^2$	$(\delta_{AB})_{13}$
$\Delta m_{B_s}$	$\sim (V_{ts}^* V_{tb})^2$	$(\delta_{AB})_{23}$
$A_{CP}(B \rightarrow \psi K_S)$	$= \sin 2\beta$	$(\delta_{AB})_{13}$
$A_{CP}(B \rightarrow \phi K_S)$	$= \sin 2\beta$	$(\delta_{AB})_{23}$

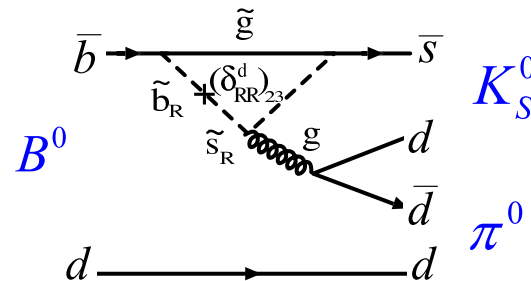
# Feynman Diagrams for $\sin 2\beta$ from Penguins $\sin 2\beta$ and... and....



In SM interference between  $B$  mixing,  $K$  mixing and Penguin  $b \rightarrow s\bar{s}s$  or  $b \rightarrow s\bar{d}d$  gives the same  $e^{-2i\beta}$  as in tree process  $b \rightarrow c\bar{c}s$ . However loops can also be sensitive to New Physics!



## New phases from SUSY?



## Introduction to CP Violation in $B$ Decays & Mixings -1

- CKM Violation in the SM is due to the phase  $\eta$  in the CKM matrix
- All CP violations in the SM are due to the interference of two *different* amplitudes

### Three Classes of CP Violation in $B$ Decays

1. Direct CP Violation: e.g., in  $B^\pm \rightarrow K_s \pi^\pm$  decays

$$\mathcal{A}_{\text{CP}} = \frac{2|A_1||A_2| \sin(\delta_1 - \delta_2) \sin(\phi_1 - \phi_2)}{|A_1|^2 + 2|A_1||A_2| \cos(\delta_1 - \delta_2) \cos(\phi_1 - \phi_2) + |A_2|^2}$$

- Requires Strong phase difference  $(\delta_1 - \delta_2) \neq 0$  & Weak phase difference  $(\phi_1 - \phi_2) \neq 0$ ; Difficult to calculate

2. Indirect CP Violation:

$$\mathcal{A}_{\text{SL}} = \frac{\Gamma(\overline{B}^0 \rightarrow \ell^+ X) - \Gamma(B^0 \rightarrow \ell^- X)}{\Gamma(\overline{B}^0 \rightarrow \ell^+ X) + \Gamma(B^0 \rightarrow \ell^- X)}$$

- Involves the relative phase in the absorptive & dispersive parts of the  $B^0 - \overline{B}^0$

Mixing Amplitude:  $\mathcal{A}_{\text{SL}} = \text{Im}\left(\frac{\Gamma_{12}}{M_{12}}\right)$

- Writing  $\left(\frac{\Gamma_{12}}{M_{12}}\right)_q = r_q e^{i\zeta_q}$ :  $\mathcal{A}_{\text{SL}}(B_d) = r_d \sin \zeta_d$

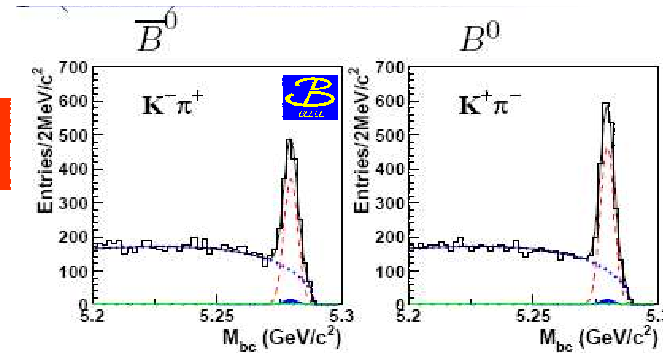
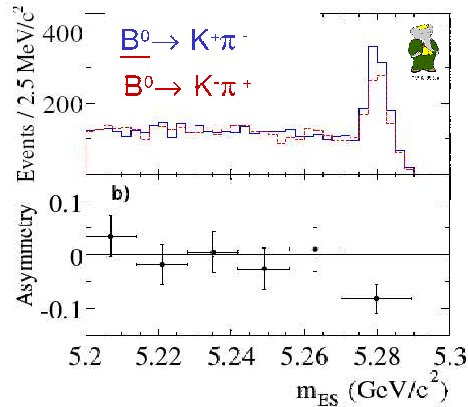
- SM (NLO):  $\mathcal{A}_{\text{SL}}(B_d) = -(5.5 \pm 1.3)(10^{-4})$  [Beneke et al.; Ciuchini et al.]

- Present Experimental Constraint:  $\mathcal{A}_{\text{SL}}(B_d) = (-0.05 \pm 0.71) \times 10^{-2}$

# Direct CP-asymmetry in $B \rightarrow K\pi$ [EPS 2005]

## Direct CP violation

For two-body decays : only seen clearly ( $> 4\sigma$ ) in one single channel



Experiment	$A_{K\pi}$	# $\sigma$
BaBar ( $227 \times 10^6$ BB)	$-0.133 \pm 0.030_{\text{stat}} \pm 0.009_{\text{syst}}$	4.2
BELLE ( $386 \times 10^6$ BB)	$-0.113 \pm 0.022_{\text{stat}} \pm 0.008_{\text{syst}}$	$\sim 5$

For three-body decays : first sign of direct CPV

$$A_{CP}(\rho^0 K^\pm) = (30 \pm 11_{\text{stat}} \pm 11_{\text{syst+model}})\% \quad \text{BELLE } (386 \times 10^6 \text{ BB})$$

Full Dalitz  $K\pi\pi$  fit

Direct CP violation at 3.9

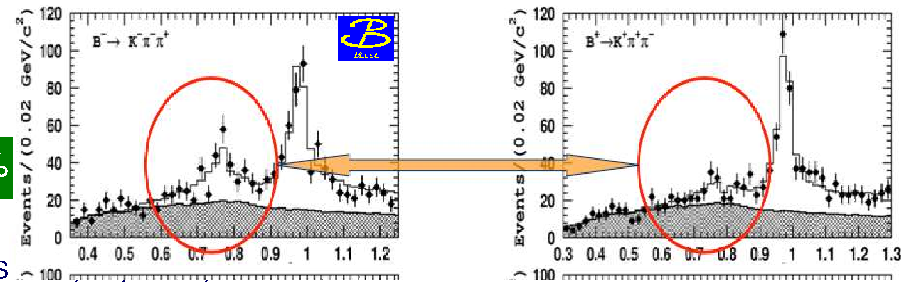
$$A_{CP}(\rho^0 K^\pm) = (34 \pm 13_{\text{stat}} \pm 6_{\text{syst}} \pm 15_{\text{model}})\%$$

BABAR  $227 \cdot 10^6$  BB

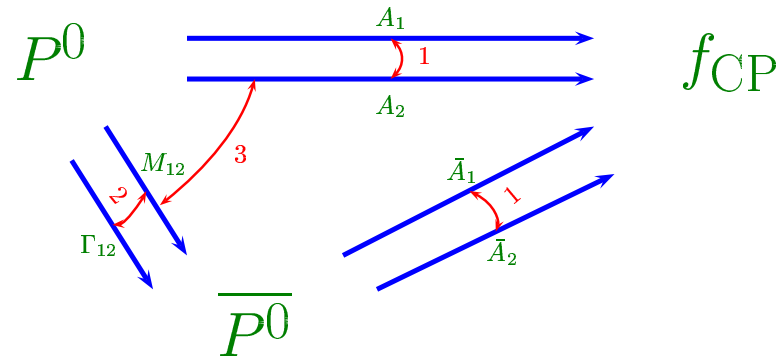


HEP-EPS

Fit projections in the  $\rho$  region



## CP violation in neutral meson decay into a CP eigenstate



1. In decay:  $\bar{A}/A \neq 1$   $\left( \frac{\bar{A}}{A} = \frac{\bar{A}_1 + \bar{A}_2}{A_1 + A_2} \right)$   
(For example,  $A_1$  is a Tree amplitude &  $A_2$  is Penguin)
2. In mixing:  $|q/p| \neq 1$   $\left( \frac{q}{p} = \frac{2M_{12}^* - i\Gamma_{12}^*}{\Delta m - (i/2)\Delta\Gamma} \right)$
3. In interference:  $\text{Im}\lambda \neq 1$   $\left( \lambda = \frac{q}{p} \frac{\bar{A}}{A} \right)$ 
  - The case theorists love!
    - Decay dominated by a single CPV phase:  $|\frac{\bar{A}}{A}| = 1$ ;
    - CPV in mixing negligible  $|\frac{q}{p}| = 1$ ;
    - The remaining effect is:  $S_f \sim \sin[\arg(M_{12}) - 2 \arg(A)] = 1$

## Interplay of Mixing & Decays of $B^0$ and $\overline{B^0}$ to CP Eigenstate

- Involving tree-dominated  $B$ -decays ( $b \rightarrow c\bar{c}s$ ), such as  $B^0/\overline{B^0} \rightarrow J/\psi K_s; J/\psi K_L$

$$\mathcal{A}_f(t) = \frac{\Gamma(\overline{B^0}(t) \rightarrow f) - \Gamma(B^0(t) \rightarrow f)}{\Gamma(\overline{B^0}(t) \rightarrow f) + \Gamma(B^0(t) \rightarrow f)}$$

$$= C_f \cos(\Delta M_B t) + S_f \sin(\Delta M_B t)$$

$$C_f = \frac{(|\lambda_f|^2 - 1)}{(|\lambda_f|^2 + 1)}; \quad S_f = \frac{2 \operatorname{Im}\lambda_f}{(|\lambda_f|^2 + 1)}$$

- Definitions:

$$\lambda_f \equiv (q/p) \rho(f); \quad \rho(f) = \frac{\bar{A}(f)}{A(f)}$$

$$A(f) = \langle f | H | B^0 \rangle; \quad \bar{A}(f) = \langle f | H | \overline{B^0} \rangle$$

$$q/p = \frac{V_{tb}^* V_{td}}{V_{td} V_{tb}^*} = e^{-2i\phi_{\text{mixing}}} = e^{-2i\beta}$$

- If only a Single Amplitude dominant, then one can write:

$$\rho(f) = \eta_f e^{-2i\phi_{\text{decay}}}$$

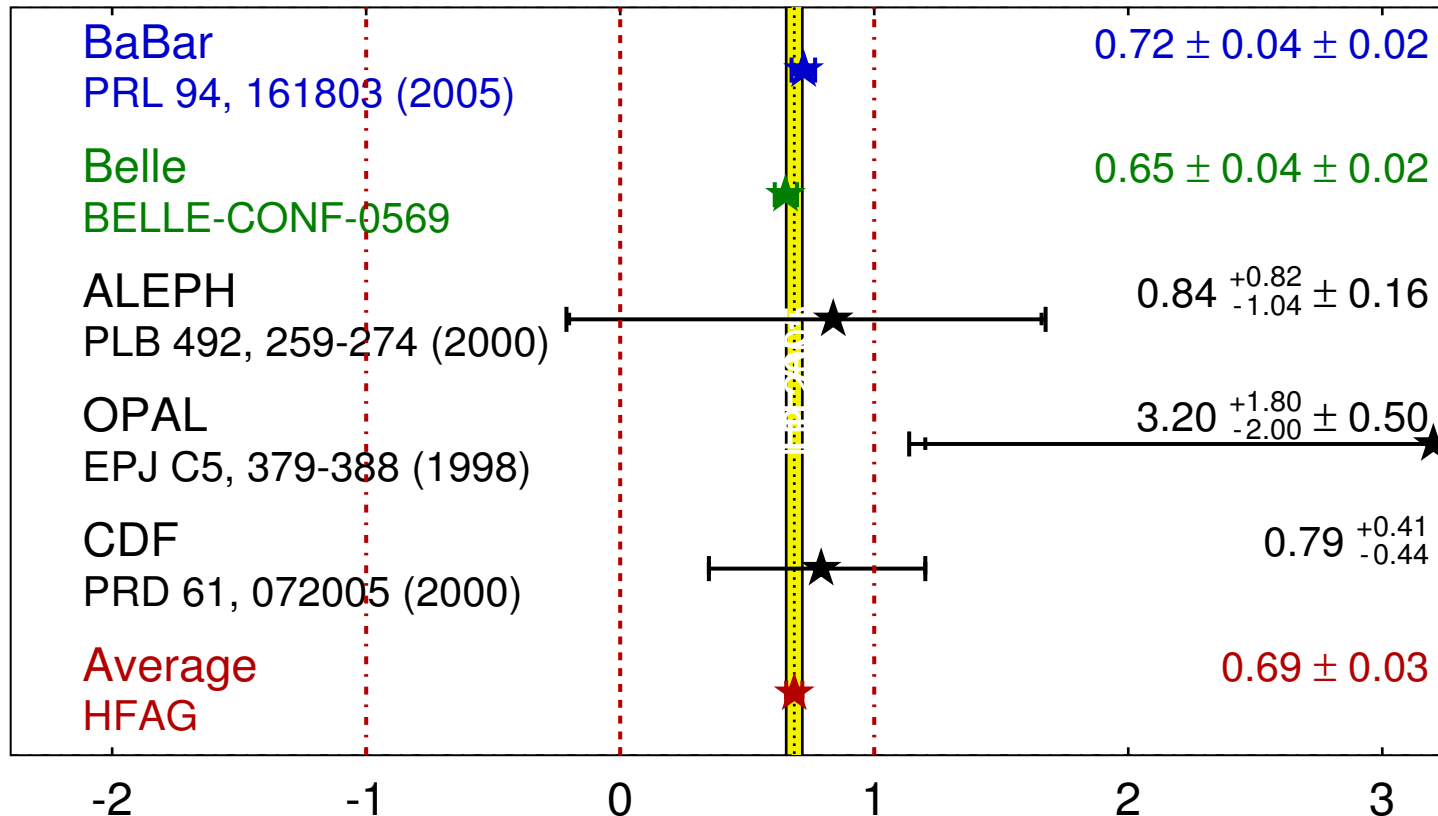
where  $\eta_f = \pm 1$  is the intrinsic CP-Parity of the state  $f \Rightarrow |\rho(f)| = 1$

$$\mathcal{A}_f(t) = S_f \sin(\Delta M_B t); \quad S_f = -\eta_f \sin 2(\phi_{\text{mixing}} + \phi_{\text{decay}}); \quad C_f = 0$$

Current World Average [EPS 2005]

$$\sin(2\beta)/\sin(2\phi_1)$$

**HFAG**  
LP 2005  
PRELIMINARY



# Current World Average [EPS 2005]

## Determination of $\beta_1$

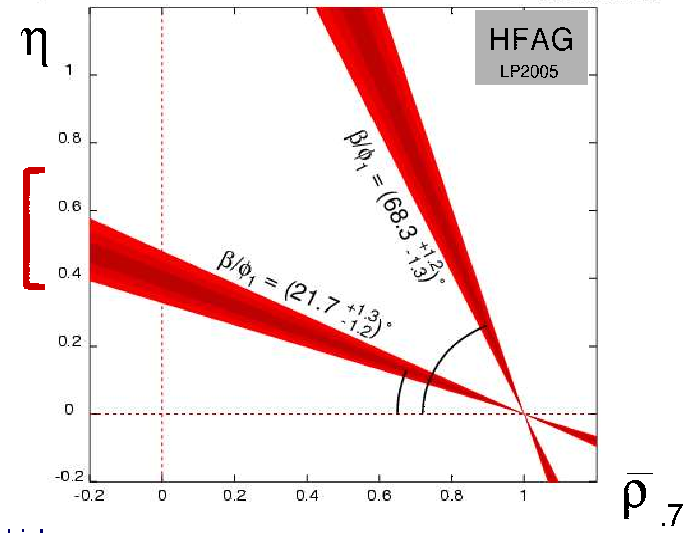
Experiment	BaBar (227 x 10 <sup>6</sup> BB)	Belle (386 x 10 <sup>6</sup> BB)
sin2 $\beta$ from (cc) K <sub>S</sub>	0.75 ± 0.04 <sub>stat</sub>	0.668 ± 0.047 <sub>stat</sub>
(cc) K <sub>L</sub>	0.57 ± 0.09 <sub>stat</sub>	0.619 ± 0.069 <sub>stat</sub>
All charmonium	0.722 ± 0.040 <sub>stat</sub> ± 0.023 <sub>syst</sub>	0.652 ± 0.039 <sub>stat</sub> ± 0.020 <sub>syst</sub>

HFAG Average : 0.685 ± 0.032    ~5% precision

Constraint from sides only:  
0.720±0.024 (CKMFitter) →

Coherent description of CPV within the SM  
SM is the dominant source of CPV

**Preferred solution:**  
Use of the B<sup>0</sup> → J/ψ K\*(→ K<sub>S</sub>π<sup>0</sup>) (VV decay) information on cos(2β) (after strong phase ambiguity resolution) BABAR PRD 71, 032005 (2005). cos(2β)>0 at 86%CL



HEP-EPS 2005, July 21-27, Lisboa

## Specific realizations of MFV Models

[Ciuchini et al.; Degrossi et al.; Carena et al.; London, AA; Buras et al.; Bartl et al.; Bobeth et al.;...]

- CKM matrix is the only source of Flavour transitions
- Effective Lagrangian in MFV models consists of the same operator basis as in the SM (except for Higgs-induced operators in large- $\tan\beta$ -regime)

$$\mathcal{H}_{\text{eff}}(\Delta F = 2) = -\frac{G_F^2 m_W^2}{(2\pi)^2} (V_{tq}^* V_{tq})^2 [C_1(Q)\mathcal{O}_1 + C_2(Q)\mathcal{O}_2 + C_3(Q)\mathcal{O}_3]$$

$$\mathcal{O}_1(|\Delta B| = 2) = \mathcal{O}_1^{\text{SM}} = \bar{d}_L^\alpha \gamma_\mu b_L^\alpha \cdot \bar{d}_L^\beta \gamma^\mu b_L^\beta$$

$$\mathcal{O}_2(|\Delta B| = 2) = \bar{d}_L^\alpha b_R^\alpha \bar{d}_L^\beta b_R^\beta$$

$$\mathcal{O}_3(|\Delta B| = 2) = \bar{d}_L^\alpha b_R^\beta \bar{d}_L^\beta b_R^\alpha$$

- SUSY effects are encoded in Wilson coefficients or, equivalently, in terms of a limited set of Inami-Lim functions which depend on the SUSY parameters

$$C_1(m_W) = C_1^W(m_W) + C_1^X(m_W, m_\chi) + C_1^H(m_W, m_H)$$

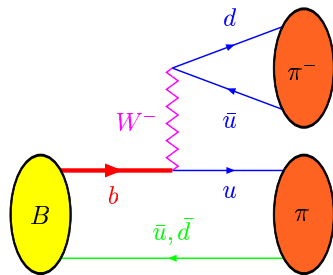
$$C_2(m_W) \propto m_b^2/m_W^2; \quad C_3(m_W) \propto m_b^2/(m_W^2 \cos^2 \beta)$$

- Typically,  $C_2(m_W)/C_1(m_W), C_3(m_W)/C_1(m_W) \ll 1$
- In constrained MSSM models,  $C_1(m_W)^{\text{SUSY}}/C_1(m_W)^{\text{SM}} = 1 + \delta$ , with  $\delta \ll 1 \implies$  UT(MFV) similar to UT(SM)

# $B \rightarrow \pi\pi$ Topologies

## Dominant topologies contributed within the Standard Model

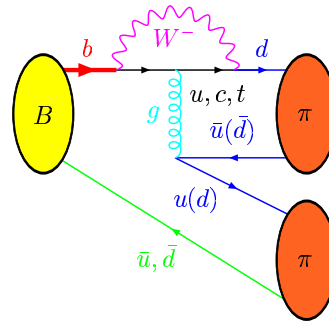
Tree ( $\mathcal{T}$ )



$$B^- \rightarrow \pi^- \pi^0$$

$$\bar{B}^0 \rightarrow \pi^+ \pi^-$$

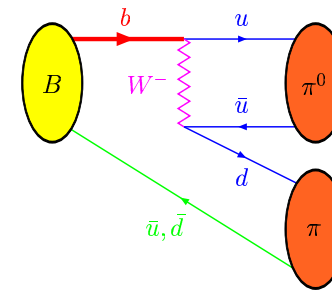
Penguin ( $\mathcal{P}$ )



$$\bar{B}^0 \rightarrow \pi^+ \pi^-$$

$$\bar{B}^0 \rightarrow \pi^0 \pi^0$$

Color-suppressed ( $\mathcal{C}$ )



$$B^- \rightarrow \pi^- \pi^0$$

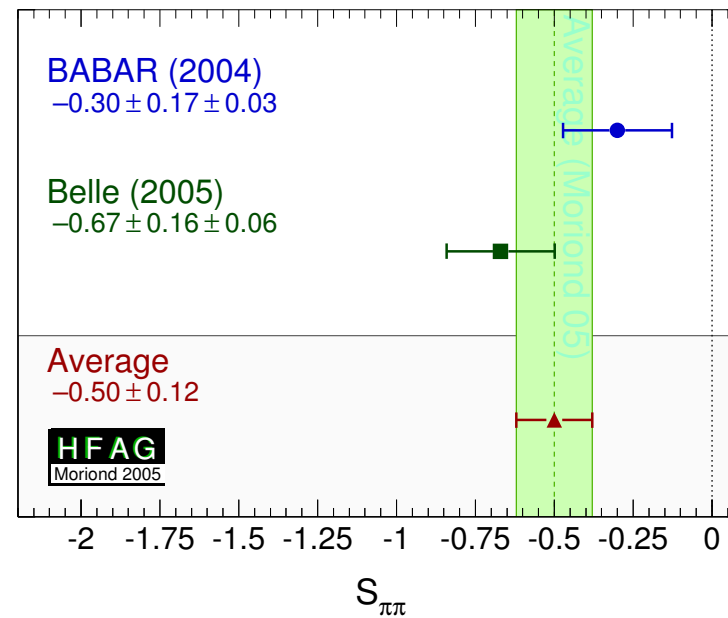
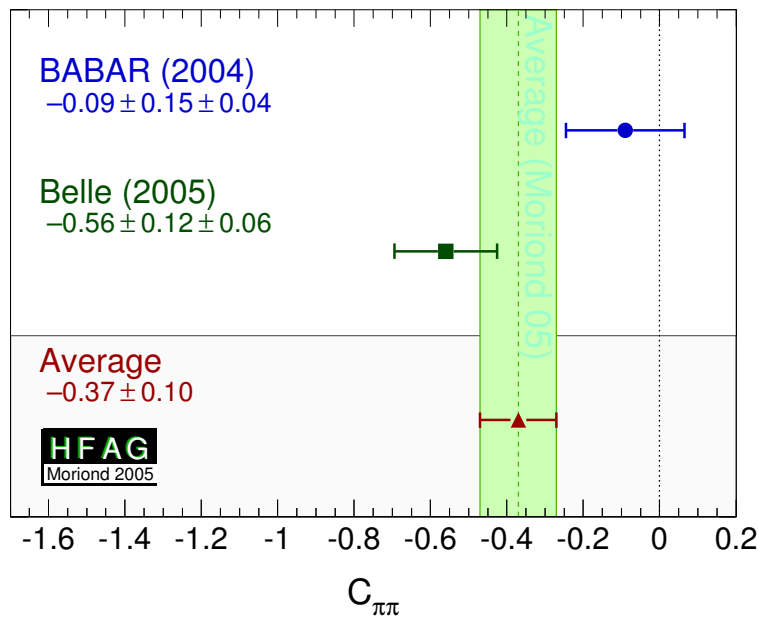
$$\bar{B}^0 \rightarrow \pi^0 \pi^0$$

## Subdominant topologies:

- exchange ( $\mathcal{E}$ )  $\implies \bar{B}^0 \rightarrow \pi^+ \pi^-, \pi^0 \pi^0$
- annihilation ( $\mathcal{A}$ )  $\implies B^- \rightarrow \pi^- \pi^0$
- penguin-annihilation ( $\mathcal{PA}$ )  $\implies \bar{B}^0 \rightarrow \pi^+ \pi^-, \pi^0 \pi^0$
- electroweak-penguin ( $\mathcal{P}_{EW}$ )  $\implies B^- \rightarrow \pi^- \pi^0, \bar{B}^0 \rightarrow \pi^0 \pi^0$
- color-suppressed electroweak-penguin ( $\mathcal{P}_{EW}^C$ )  $\implies \bar{B}^0 \rightarrow \pi^+ \pi^-, \pi^0 \pi^0$

# Time-dependent CP-asymmetry in $B^0/\bar{B}^0 \rightarrow \pi^+\pi^-$

$$\begin{aligned}
 a_{\pi\pi}^{+-}(t) &\equiv \frac{\Gamma[\bar{B}^0(t) \rightarrow \pi^+\pi^-] - \Gamma[B^0(t) \rightarrow \pi^+\pi^-]}{\Gamma[\bar{B}^0(t) \rightarrow \pi^+\pi^-] + \Gamma[B^0(t) \rightarrow \pi^+\pi^-]} \\
 &= S_{\pi\pi}^{+-}(t) \sin(\Delta M_B t) - C_{\pi\pi}^{+-}(t) \cos(\Delta M_B t)
 \end{aligned}$$



- BELLE and BABAR are in better agreement now ( $2.3\sigma$ )

## $C_{\pi\pi} - S_{\pi\pi}$ Data vs. QCDF & PQCD

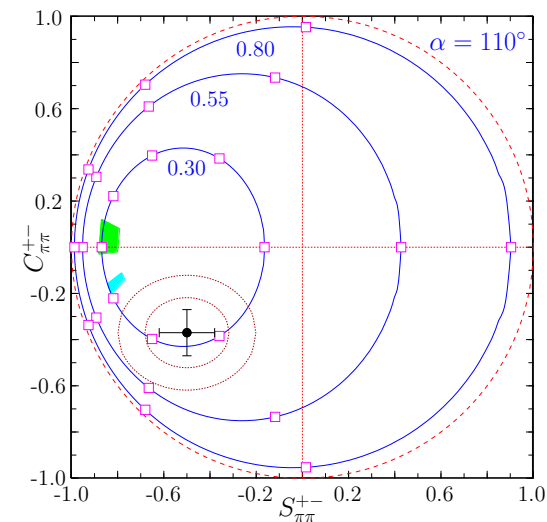
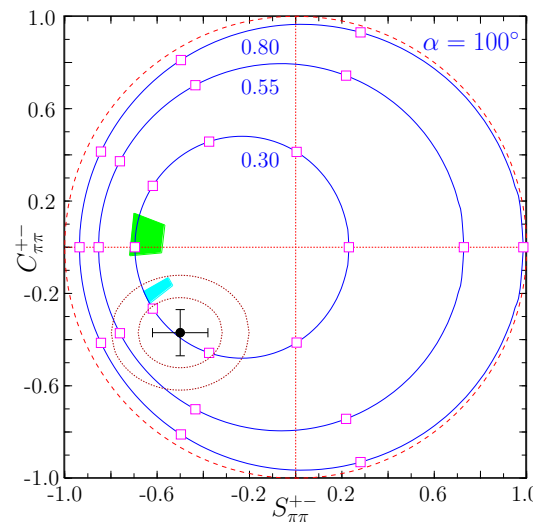
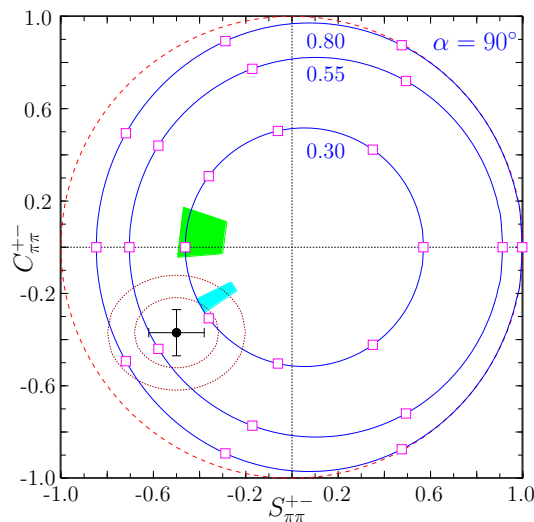
- Isospin-based analysis of data (68% C.L.)

[Lunghi, Parkhomenko, AA; Parkhomenko '05]

$$|P/T| = 0.48_{-0.08}^{+0.09} \quad \delta = (-38_{-9.0}^{+9.3})^\circ$$

- Similar analyses by Buras et al.; Bauer et al.; Gronau et al.; ...
- **Theoretical predictions based on Factorization**

$$\begin{array}{lll} |P/T| = 0.29 \pm 0.09 & \delta = (9 \pm 15)^\circ & \text{QCDF} \\ |P/T| = 0.23_{-0.05}^{+0.07} & \delta = (-37 \pm 5)^\circ & \text{PQCD} \end{array}$$



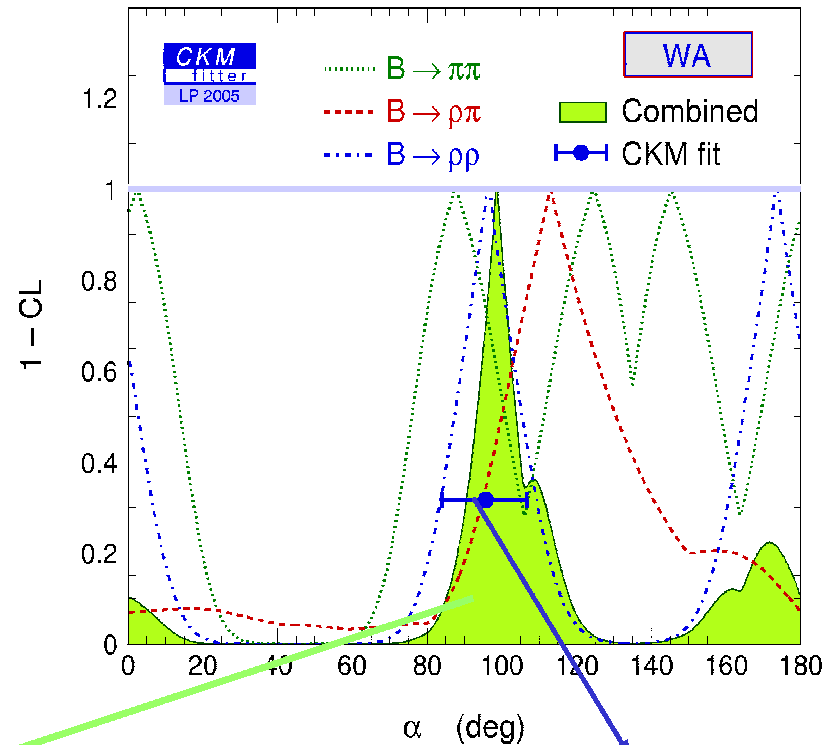
- Data supports **large** penguin contribution and **strong phase difference**

# Current World Average of $\alpha$ [EPS 2005]

## Overall constraint on $\alpha$

Additional constraints on  $\alpha$  from time dependent CP violation analysis of  $B \rightarrow \rho\pi$

- $\pi\pi$  determination very weak
- $\rho\rho$  best individual measurement
- Mirror solution are disfavored by  $\rho\pi$



$$\alpha = (99^{+12}_{-9})^\circ$$

Same precision as global CKM fit :  
 $\alpha_{CKM} = (94.9^{+9.6}_{-13.3})^\circ$



## Current World Average of $\gamma$ [EPS 2005]

# Overall information on $r_B$ and $\phi_3/\gamma$ from GLW, ADS, Dalitz

$$r_{DK} = 0.081 \pm 0.029 [0.021, 0.138] @95\% \text{ CL}$$

$$r_{D^*K} = 0.088 \pm 0.042 [0.010, 0.170] @95\% \text{ CL}$$

$$r_{DK^*} = 0.15 \pm 0.09 [0.01, 0.31] @95\% \text{ CL}$$

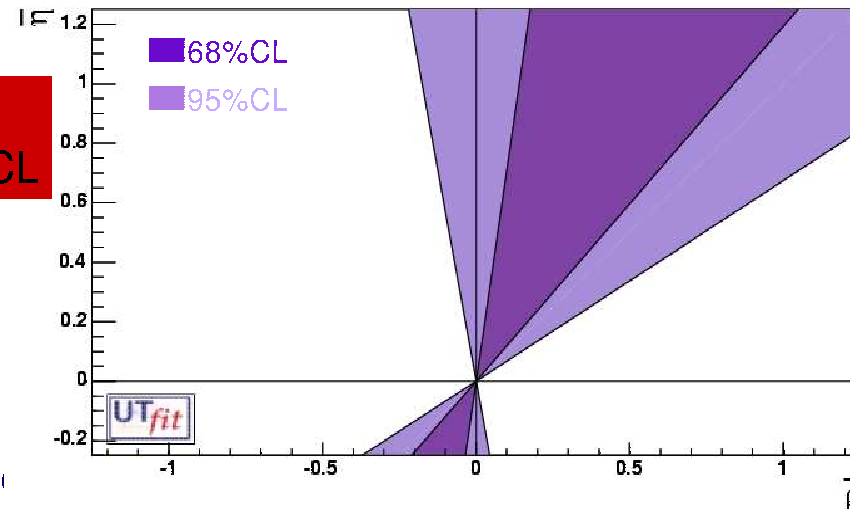
UTfit

$\Rightarrow r_B$  is small

$\Rightarrow \gamma$  is not easy to measure...

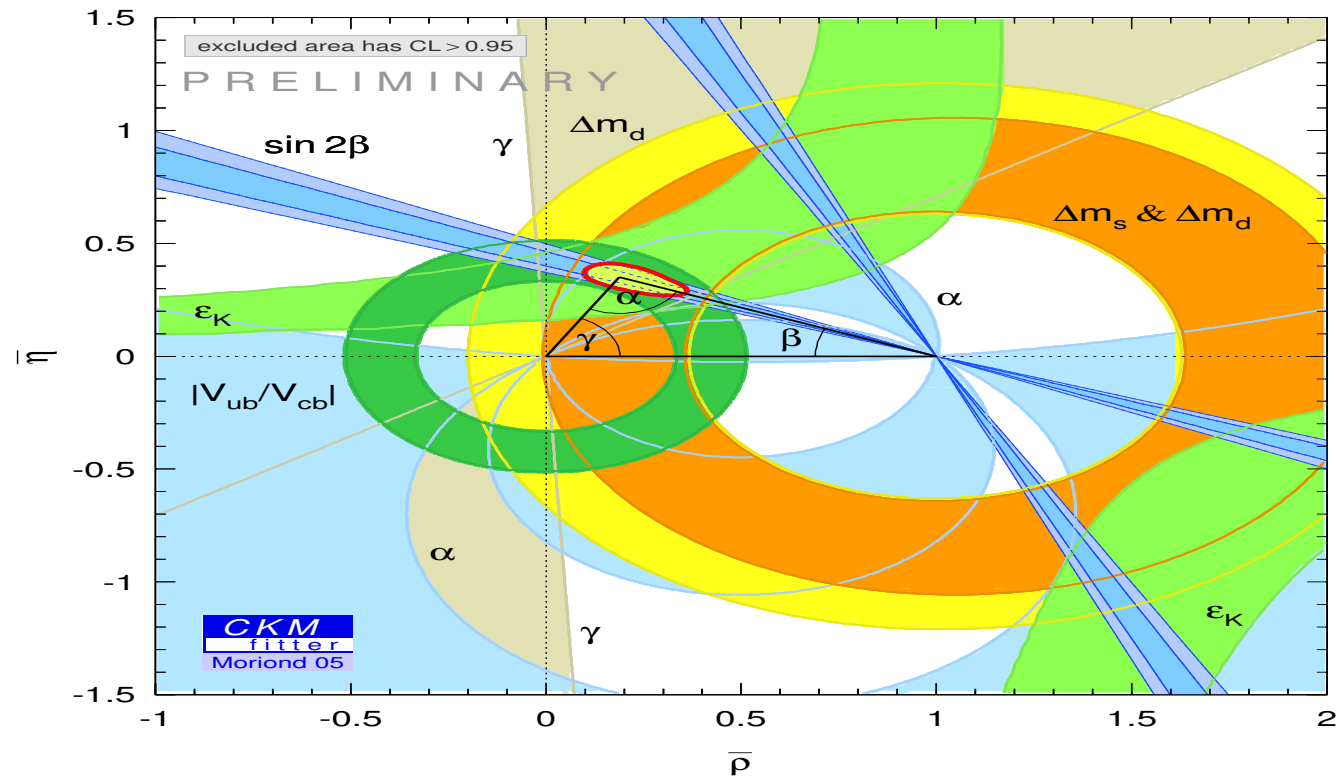
$$\gamma = 66 \pm 17 [32, 102] \text{ at } 95\% \text{ CL}$$

$$\gamma = -114 \pm 17 [-148, -78] \text{ at } 95\% \text{ CL}$$



HEP-EPS 2005

## SM confronts measurements of $\sin 2\beta$ , $\alpha$ , $\gamma$

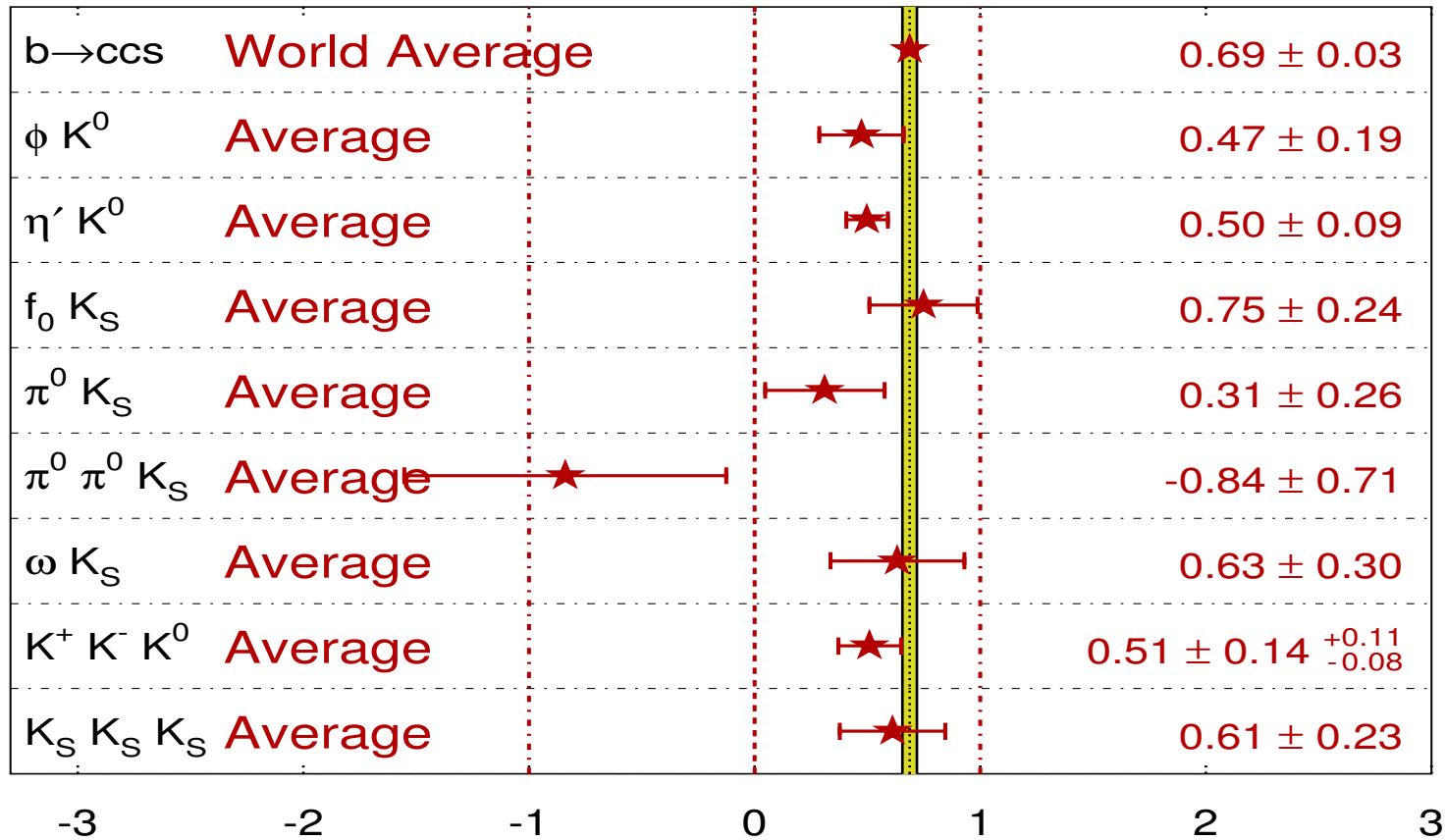


- $\sin 2\beta = 0.685 \pm 0.032 (\beta = [21.7 \pm 1.3]^\circ)$
- $\alpha = [99_{-9}^{+12}]^\circ$
- $\gamma = [66 \pm 17]^\circ$
- Direct and indirect measurements of angles agree very well
- Unconstrained sum of angles =  $(187 \pm 21)^\circ$ , consistent with unitarity sum within errors

$S_{b \rightarrow q\bar{q}s}$  and  $C_{b \rightarrow q\bar{q}s}$  [HFAG 2005; hep-ex/0505100]

$$\sin(2\beta^{\text{eff}})/\sin(2\phi_1^{\text{eff}})$$

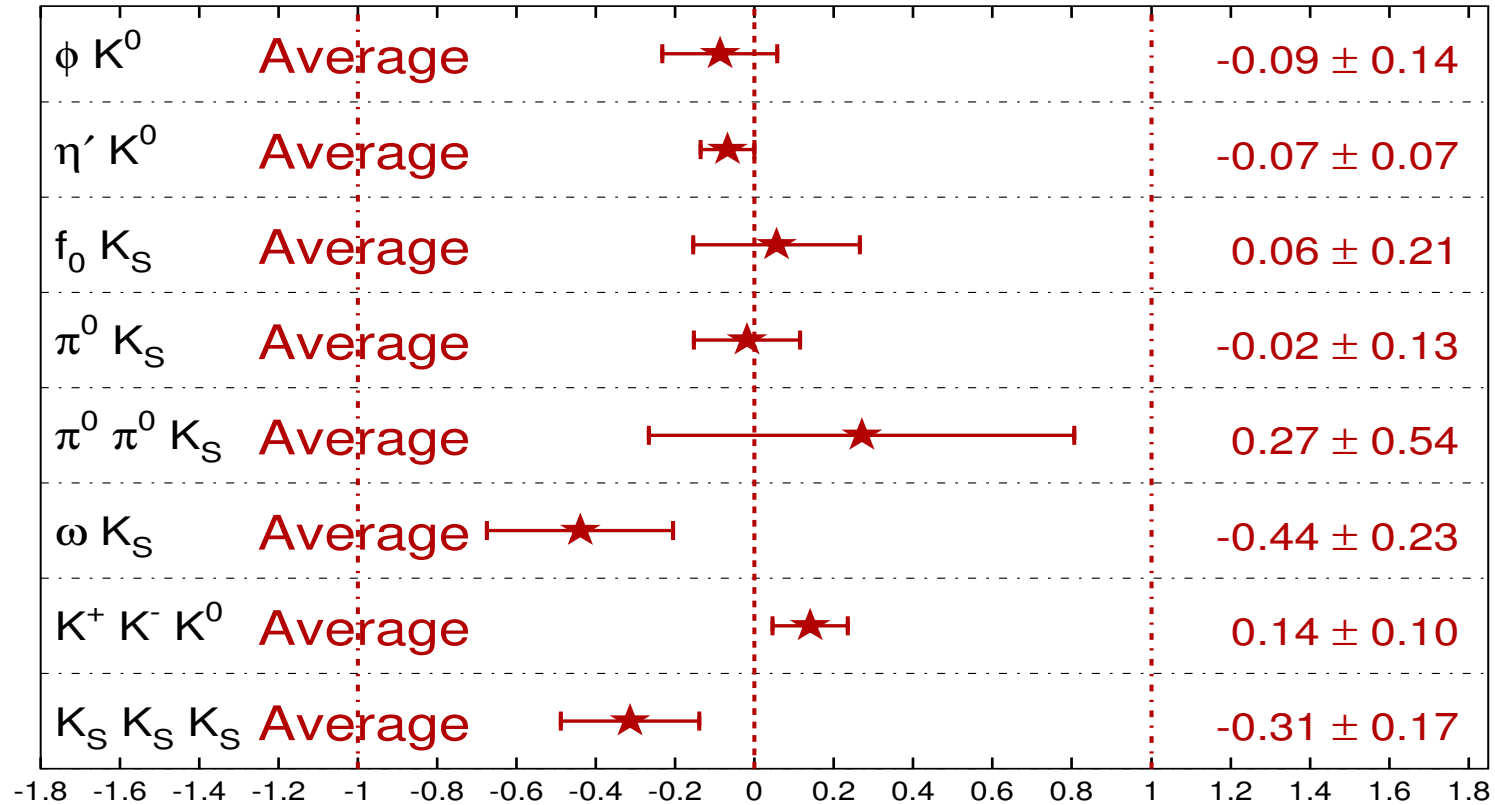
**HFAG**  
HEP 2005  
PRELIMINARY



$S_{b \rightarrow q\bar{q}s}$  and  $C_{b \rightarrow q\bar{q}s}$  [HFAG 2005; hep-ex/0505100]

$$C_f = -A_f$$

**HFAG**  
HEP 2005  
PRELIMINARY



## The effective Lagrangian:

$$\mathcal{L} = \mathcal{L}_{QCD \times QED}(q, l) + \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{10} C_i(\mu) O_i$$

$(q = u, d, s, c, b, \quad l = e, \mu)$

$$O_i = \left\{ \begin{array}{lll} (\bar{s}\Gamma_i c)(\bar{c}\Gamma'_i b), & i = 1, 2, & |C_i(m_b)| \sim 1 \\ (\bar{s}\Gamma_i b)\Sigma_q(\bar{q}\Gamma'_i q), & i = 3, 4, 5, 6, & |C_i(m_b)| < 0.07 \\ \frac{em_b}{16\pi^2} \bar{s}_L \sigma^{\mu\nu} b_R F_{\mu\nu}, & i = 7, & C_7(m_b) \sim -0.3 \\ \frac{gm_b}{16\pi^2} \bar{s}_L \sigma^{\mu\nu} T^a b_R G_{\mu\nu}^a, & i = 8, & C_8(m_b) \sim -0.15 \\ \frac{e^2}{16\pi^2} (\bar{s}_L \gamma_\mu b_L)(\bar{l} \gamma^\mu \gamma_5 l), & i = 9, \mathbf{10} & |C_i(m_b)| \sim 4 \end{array} \right.$$

Three steps of the calculation:

**Matching:** Evaluating  $C_i(\mu_0)$  at  $\mu_0 \sim M_W$  by requiring equality of the SM and the effective theory Green functions

**Mixing:** Deriving the effective theory RGE and evolving  $C_i(\mu)$  from  $\mu_0$  to  $\mu_b \sim m_b$

**Matrix elements:** Evaluating the on-shell amplitudes at  $\mu_b \sim m_b$

## $\bar{B} \rightarrow X_s l^+ l^-$

- The NNLO calculation of  $\bar{B} \rightarrow X_s l^+ l^-$  corresponds to the NLO calculation of  $\bar{B} \rightarrow X_s \gamma$ , as far as the number of loops in the diagrams is concerned.
- Coefficients of the two additional operators

$$O_i = \frac{e^2}{16\pi^2} (\bar{s}_L \gamma_\mu b_L) (\bar{l} \gamma^\mu \gamma_5 l), \quad i = 9, 10$$

have the following perturbative expansion:

$$C_9(\mu) = \frac{4\pi}{\alpha_s(\mu)} C_9^{(-1)}(\mu) + C_9^{(0)}(\mu) + \frac{\alpha_s(\mu)}{4\pi} C_9^{(1)}(\mu) + \dots$$

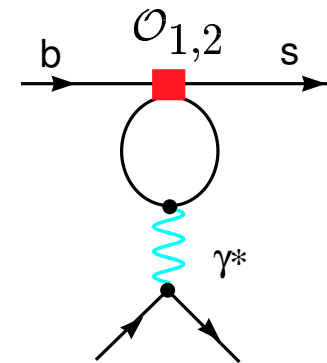
$$C_{10} = C_{10}^{(0)} + \frac{\alpha_s(M_W)}{4\pi} C_{10}^{(1)} + \dots$$

- After an expansion in  $\alpha_s$ , the term  $C_9^{(-1)}(\mu)$  reproduces (the dominant part of) the electro-weak logarithm that originates from photonic penguins with charm quark loops:

$$\frac{4\pi}{\alpha_s(m_b)} C_9^{(-1)}(m_b) = \frac{4}{9} \ln \frac{M_W^2}{m_b^2} + \mathcal{O}(\alpha_s)$$

$$C_9^{(-1)}(m_b) \simeq 0.033 \ll 1 \quad \Rightarrow \quad \frac{4\pi}{\alpha_s(m_b)} C_9^{(-1)}(m_b) \simeq 2$$

$$\text{On the other hand:} \quad C_9^{(0)}(m_b) \simeq 2.2$$



## NNLO Calculations of $\text{BR}(\bar{B} \rightarrow X_s \ell^+ \ell^-)$

- Two-loop matching, three-loop mixing and two-loop matrix elements have been completed
  - Matching: [Bobeth, Misiak, Urban]
  - Mixing: [Gambino, Gorbahn, Haisch]
  - Matrix elements:  
[Asatryan, Asatrian, Greub, Walker;  
Asatrian, Bieri, Greub, Hovhannissyan;  
Ghinculov, Hurth, Isidori, Yao;  
Bobeth, Gambino, Gorbahn, Haisch]
- Power corrections in  $B \rightarrow X_s \ell^+ \ell^-$  decays
  - $1/m_b$  corrections [A. Falk et al.; AA, Handoko, Morozumi, Hiller; Buchalla, Isidori]
  - $1/m_c$  corrections [Buchalla, Isidori, Rey]
- NNLO Phenomenological analysis of  $B \rightarrow X_s \ell^+ \ell^-$  decays  
[AA, Greub, Hiller, Lunghi]
  - $\text{BR}(\bar{B} \rightarrow X_s \mu^+ \mu^-); \quad q^2 > 4m_\mu^2 = (4.2 \pm 1.0) \times 10^{-6}$
  - $\text{BR}(\bar{B} \rightarrow X_s e^+ e^-) = (6.9 \pm 0.7) \times 10^{-6}$

## Electroweak Penguins $b \rightarrow s\ell^+\ell^-$

- $B \rightarrow X_s\ell^+\ell^-$  decay rate

$$\mathcal{B}(B \rightarrow X_s\ell^+\ell^-) = (4.46_{-0.96}^{+0.98}) \times 10^{-6} \quad [\text{HFAG}'05]$$

$$SM : (4.2 \pm 0.7) \times 10^{-6} \quad [\text{AGHL}'01]; \quad (4.6 \pm 0.8) \times 10^{-6} \quad [\text{GHIY}'04]$$

- Differential distributions in  $B \rightarrow X_s\ell^+\ell^-$

- $M(X_s)$ -distribution: tests  $s \rightarrow X_s$  fragmentation model; current FMs provide reasonable fit to data

- $q^2 = M_{\ell^+\ell^-}^2$ -distribution away from the  $J/\psi, \psi', \dots$  resonances is sensitive to short-distance physics; current data in agreement with the SM estimates but the precision is not better than 25%

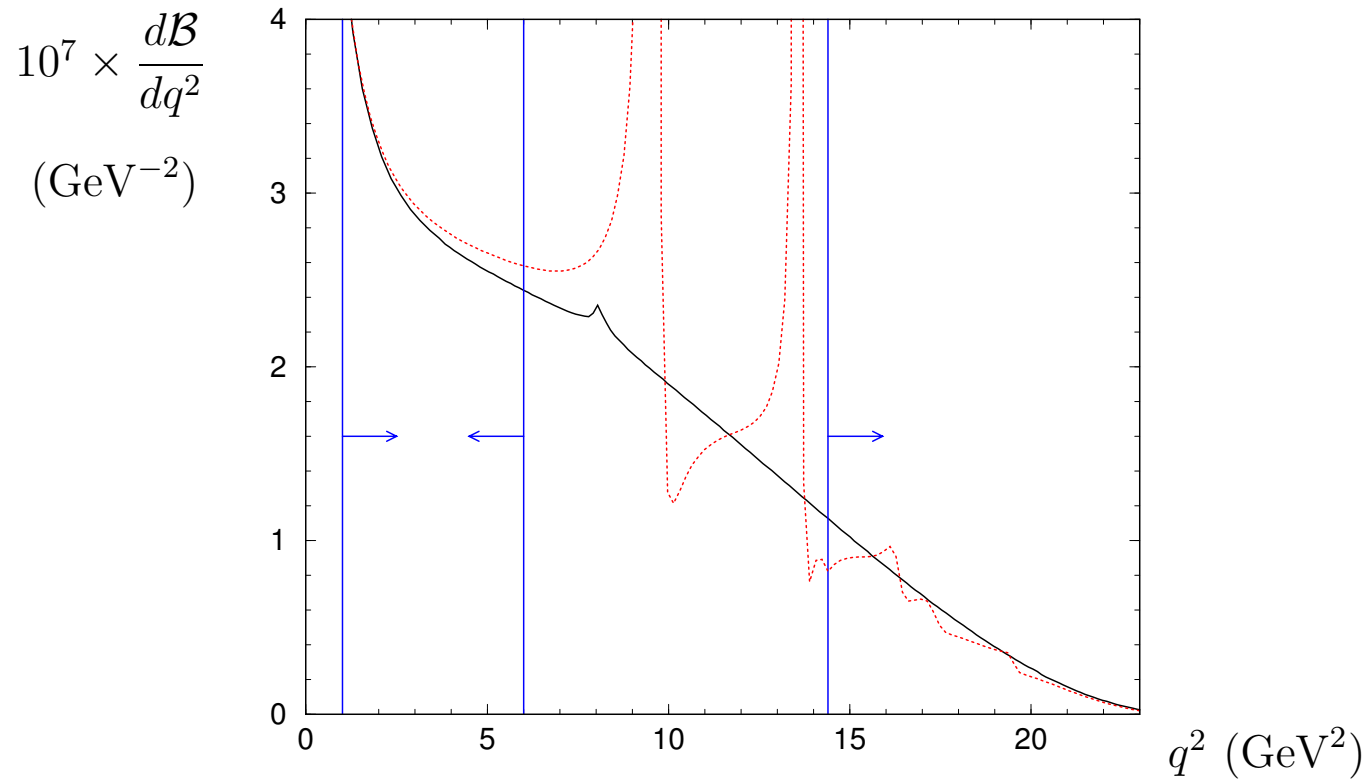
- Forward-Backward Asymmetry (FBA) is likewise sensitive to the SM and BSM effects, in particular encoded in the Wilson coefficients  $C_7, C_9$  and  $C_{10}$

$$A_{\text{FB}}(\hat{s}) \sim C_{10}(2C_7 + C_9(\hat{s})\hat{s}); \quad \hat{s} = q^2/M_B^2$$

- $A_{\text{FB}}(\hat{s})$  not yet measured; possible only in experiments at  $B$  factories

## Dilepton invariant mass distribution in $\bar{B} \rightarrow X_s \ell^+ \ell^-$ :

[Ghinculov, Hurth, Isidori, Yao 2004]



- $\text{BR}(\bar{B} \rightarrow X_s \ell^+ \ell^-); \quad q^2 \in [1, 6] \text{ GeV}^2 = (1.63 \pm 0.20) \times 10^{-6}$
- $\text{BR}(\bar{B} \rightarrow X_s \ell^+ \ell^-); \quad q^2 > 14 \text{ GeV}^2 = (4.04 \pm 0.78) \times 10^{-7}$
- $\text{BR}(\bar{B} \rightarrow X_s \mu^+ \mu^-); \quad q^2 > 4m_\mu^2 = (4.6 \pm 0.8) \times 10^{-6}$ ,  
in agreement with the earlier NNLO analysis

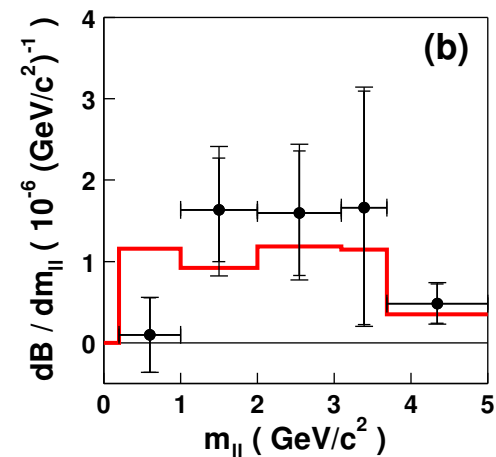
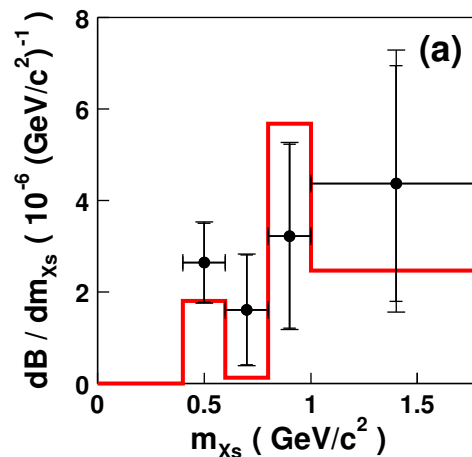
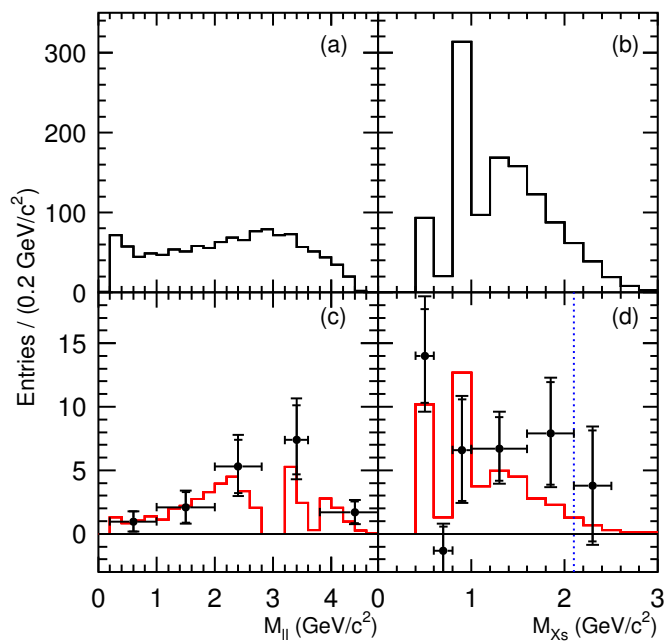
[AA, Greub, Hiller, Lunghi 2001; Bobeth, Gambino, Gorbahn, Haisch, 2003]

# Decay distributions in $\bar{B} \rightarrow X_s \ell^+ \ell^-$

## $M_{\ell\ell}$ and $M_{X_s}$ Spectra

[BELLE]

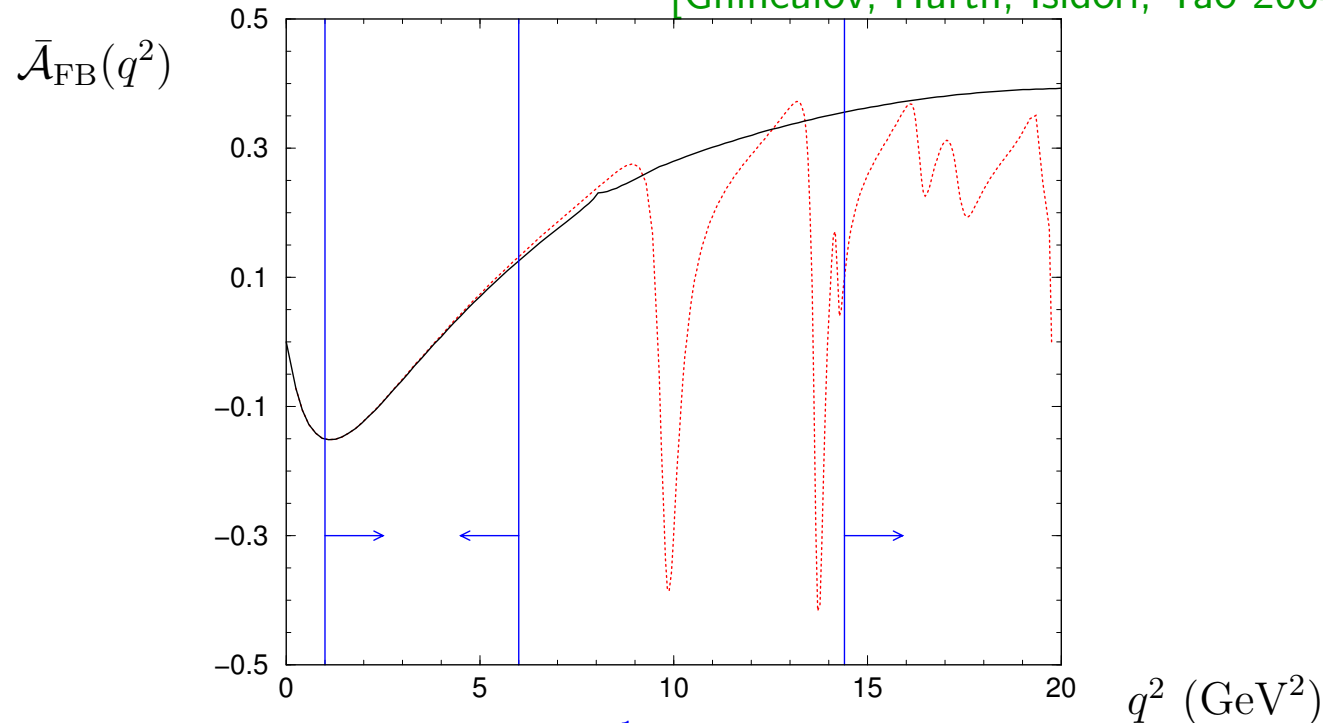
[BABAR]



- In agreement with the NNLO SM calculations

## Normalized FB-Asymmetry in $\bar{B} \rightarrow X_s \ell^+ \ell^-$ :

[Ghinculov, Hurth, Isidori, Yao 2004]



$$\bar{\mathcal{A}}_{\text{FB}}(q^2) = \frac{1}{d\mathcal{B}(B \rightarrow X_s \ell^+ \ell^-)/dq^2} \int_{-1}^1 d \cos \theta_\ell \frac{d^2 \mathcal{B}(B \rightarrow X_s \ell^+ \ell^-)}{dq^2 d \cos \theta_\ell} \text{sgn}(\cos \theta_\ell)$$

- Zero of the FB-Asymmetry is a precision test of the SM

$$q_0^2 = (3.90 \pm 0.25) \text{ GeV}^2 \quad [\text{Ghinculov, Hurth, Isidori, Yao 2004}]$$

$$q_0^2 = (3.76 \pm 0.22_{\text{theory}} \pm 0.24_{m_b}) \text{ GeV}^2 \quad [\text{Bobeth, Gambino, Gorbahn, Haisch 2003}]$$

## Electroweak Penguins $b \rightarrow s\ell^+\ell^-$

- $B \rightarrow (K, K^*)\ell^+\ell^-$  decay rates

- Decay rates and distributions depend on the form factors; estimates given below based on Light-cone QCD Sum Rules [Ball, Hiller, Handoko, AA]; Several competing estimates available in the literature [Zhong et al; Melnikov et al.;...]

$$\mathcal{B}(B \rightarrow K\ell^+\ell^-) = (0.45 \pm 0.05) \times 10^{-6} \text{ [HFAG'05]}; (0.35 \pm 0.12) \times 10^{-6} \text{ [SM]}$$

$$\mathcal{B}(B \rightarrow K^*e^+e^-) = (1.26 \pm 0.28) \times 10^{-6} \text{ [HFAG'05]}; (1.6 \pm 0.5) \times 10^{-6} \text{ [SM]}$$

$$\mathcal{B}(B \rightarrow K^*\mu^+\mu^-) = (1.45 \pm 0.23) \times 10^{-6} \text{ [HFAG'05]}; (1.2 \pm 0.5) \times 10^{-6} \text{ [SM]}$$

- Differential distributions in  $B \rightarrow (K, K^*)\ell^+\ell^-$

- $q^2 = M_{\ell^+\ell^-}^2$ -distribution away from the  $J/\psi, \psi', \dots$  resonances is sensitive to short-distance physics; current data in agreement with the SM estimates but theoretical precision is not better than 35% due to FF dependence

- The ratio  $\mathcal{B}(B \rightarrow K^*\mu^+\mu^-)/\mathcal{B}(B \rightarrow K^*e^+e^-)$  sensitive to SUSY effects in the large- $\tan\beta$  region due to Higgs effects

- $A_{\text{FB}}(\hat{s})[B \rightarrow K\ell^+\ell^-] \simeq 0$  in the SM and most BSM extensions; in agreement with data which is used as a control sample to measure  $A_{\text{FB}}(\hat{s})[B \rightarrow K^*\ell^+\ell^-]$

- $A_{\text{FB}}(\hat{s})$  in  $B \rightarrow K^*\ell^+\ell^-$  qualitatively similar to  $A_{\text{FB}}(\hat{s})$  in  $B \rightarrow X_s\ell^+\ell^-$ , except for FF complication; First measurements from BELLE at hand, appear SM-like; Super-B and LHC-B will measure  $A_{\text{FB}}(\hat{s})$  precisely



# $B \rightarrow K^{(*)} l^+ l^-$

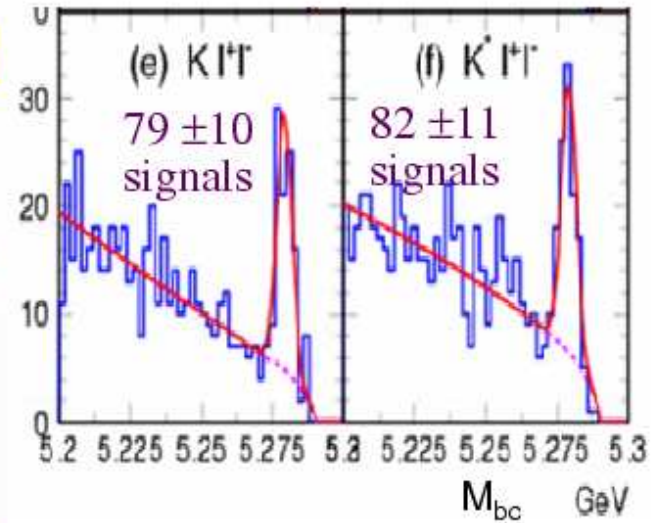
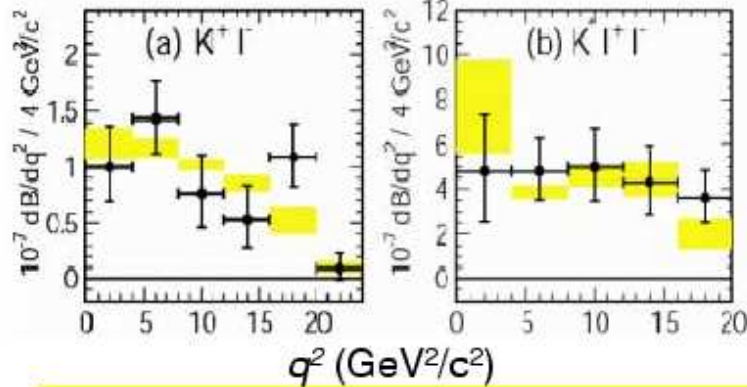
[Belle-conf-0415]

LP03:  $B \rightarrow X_s ll, K^{(*)} ll$  : Belle/BaBar  
 $Br, A_{CP} \sim SM$

**BELLE** 275M  $B\bar{B}$  update **>10 $\sigma$  signals**

$$B(Kll) = (5.50 \pm 0.75 \pm 0.27 \pm 0.02) \pm 0.70$$

$$B(K^*ll) = (16.5 \pm 2.3 \pm 0.9 \pm 0.4) \pm 2.2 \times 10^{-7}$$



# Comparison of $\mathcal{B}(B \rightarrow (K, K^*)\ell^+\ell^-)$ with SM-1

$B \rightarrow K\ell^+\ell^-$  and  $B \rightarrow K^*\ell^+\ell^-$

Belle branching fractions ( $253 \text{ fb}^{-1}$ )

-  $K\ell^+\ell^-$ :  $(5.50^{+0.75}_{-0.70} \pm 0.27 \pm 0.02) \times 10^{-7}$

-  $K^*\ell^+\ell^-$ :  $(16.5^{+2.3}_{-2.2} \pm 0.9 \pm 0.4) \times 10^{-7}$

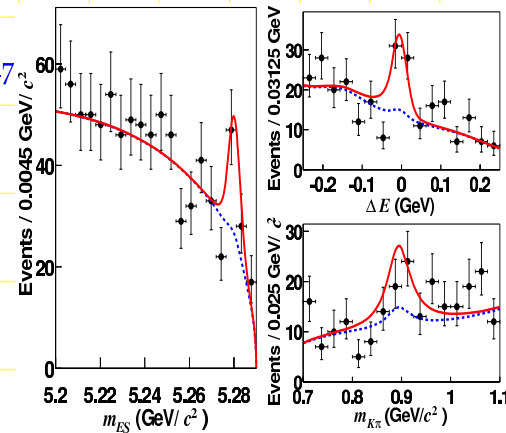
New BaBar results ( $208 \text{ fb}^{-1}$ )

-  $K\ell^+\ell^-$ :  $(3.4 \pm 0.7 \pm 0.3) \times 10^{-7}$

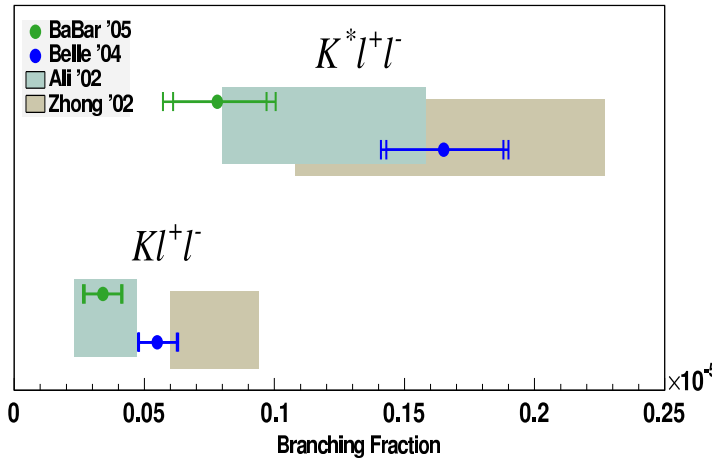
-  $K^*\ell^+\ell^-$ :  $(7.8^{+1.9}_{-1.7} \pm 1.2) \times 10^{-7}$

$A_{CP}(B^+ \rightarrow K^+\ell^+\ell^-) = -0.08 \pm 0.22 \pm 0.11$

$A_{CP}(B \rightarrow K^*\ell^+\ell^-) = +0.03 \pm 0.23 \pm 0.12$

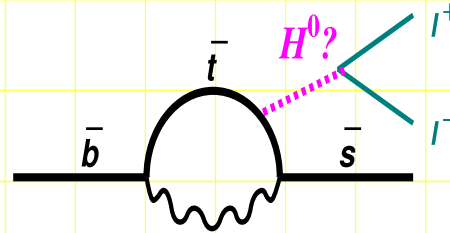
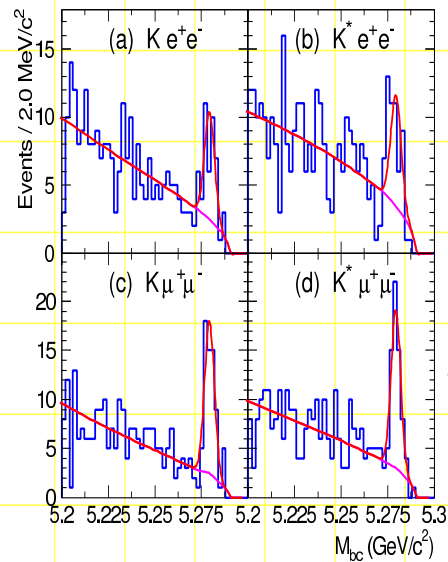


Radiative Penguins — Mikihiro Nakao — p.25



# Comparison of $\mathcal{B}(B \rightarrow (K, K^*)\ell^+\ell^-)$ with SM-2

$B \rightarrow K^{(*)}e^+e^-$  vs  $B \rightarrow K^{(*)}\mu^+\mu^-$



Ratio of  $K^{(*)}\mu^+\mu^-$  to  $K^{(*)}e^+e^-$  is sensitive to neutral SUSY Higgs if  $\tan\beta$  is large  
( $O(1)$  enhancement if  $\tan\beta \sim 30$ )

Radiative Fengluns — Mikihiro Nakao — p.26

Belle ratios:

$$\mathcal{B}(B \rightarrow K\mu^+\mu^-)/\mathcal{B}(B \rightarrow Ke^+e^-) = 1.38^{+0.39+0.06}_{-0.41-0.07} \quad (1.00 \text{ in SM})$$

$$\mathcal{B}(B \rightarrow K^*\mu^+\mu^-)/\mathcal{B}(B \rightarrow K^*e^+e^-) = 0.98^{+0.30}_{-0.31} \pm 0.08 \quad (\sim 0.75 \text{ in SM})$$

Babar ratios:

$$\mathcal{B}(B \rightarrow K\mu^+\mu^-)/\mathcal{B}(B \rightarrow Ke^+e^-) = 1.06 \pm 0.48 \pm 0.05 \quad (1.00 \text{ in SM})$$

$$\mathcal{B}(B \rightarrow K^*\mu^+\mu^-)/\mathcal{B}(B \rightarrow K^*e^+e^-) = 0.93 \pm 0.46 \pm 0.06 \quad (\sim 0.75 \text{ in SM})$$

## Forward-Backward Asymmetry in $B \rightarrow K^* \ell^+ \ell^-$

$$\frac{dA_{FB}}{d\hat{s}} = - \int_0^{\hat{u}(\hat{s})} d\hat{u} \frac{d\Gamma}{d\hat{u}d\hat{s}} + \int_{-\hat{u}(\hat{s})}^0 d\hat{u} \frac{d\Gamma}{d\hat{u}d\hat{s}}$$

$$\sim C_{10} [\text{Re}(C_9^{eff}) V A_1 + \frac{\hat{m}_b}{\hat{s}} C_7^{eff} (V T_2 (1 - \hat{m}_V) + A_1 T_1 (1 + \hat{m}_V))]$$

- $T_1, T_2, V, A_1$  form factors

- Probes different combinations of WC's than dilepton mass spectrum; has a characteristic zero in the SM ( $\hat{s}_0$ ) below  $m_{J/\psi}^2$

### Position of the $A_{FB}(\hat{s})$ zero ( $\hat{s}_0$ ) in $B \rightarrow K^* \ell^+ \ell^-$

$$\text{Re}(C_9^{eff}(\hat{s}_0)) = - \frac{\hat{m}_b}{\hat{s}_0} C_7^{eff} \left( \frac{T_2(\hat{s}_0)}{A_1(\hat{s}_0)} (1 - \hat{m}_V) + \frac{T_1(\hat{s}_0)}{V(\hat{s}_0)} (1 + \hat{m}_V) \right)$$

- Model-dependent studies  $\implies$  small FF-related uncertainties in  $\hat{s}_0$  [Burdman '98]
- HQET provides a symmetry argument why the uncertainty in  $\hat{s}_0$  is small. In leading order in  $1/m_B, 1/E$  ( $E = \frac{m_B^2 + m_{K^*}^2 - q^2}{2m_B}$ ) and  $O(\alpha_s)$ :

$$\frac{T_2}{A_1} = \frac{1 + \hat{m}_V}{1 + \hat{m}_V^2 - \hat{s}} \left( 1 - \frac{\hat{s}}{1 - \hat{m}_V^2} \right); \quad \frac{T_1}{V} = \frac{1}{1 + \hat{m}_V}$$

- No hadronic uncertainty in  $\hat{s}_0$  [AA, Ball, Handoko, Hiller '99]:

$$C_9^{eff}(\hat{s}_0) = - \frac{2m_b M_B}{s_0} C_7^{eff}$$

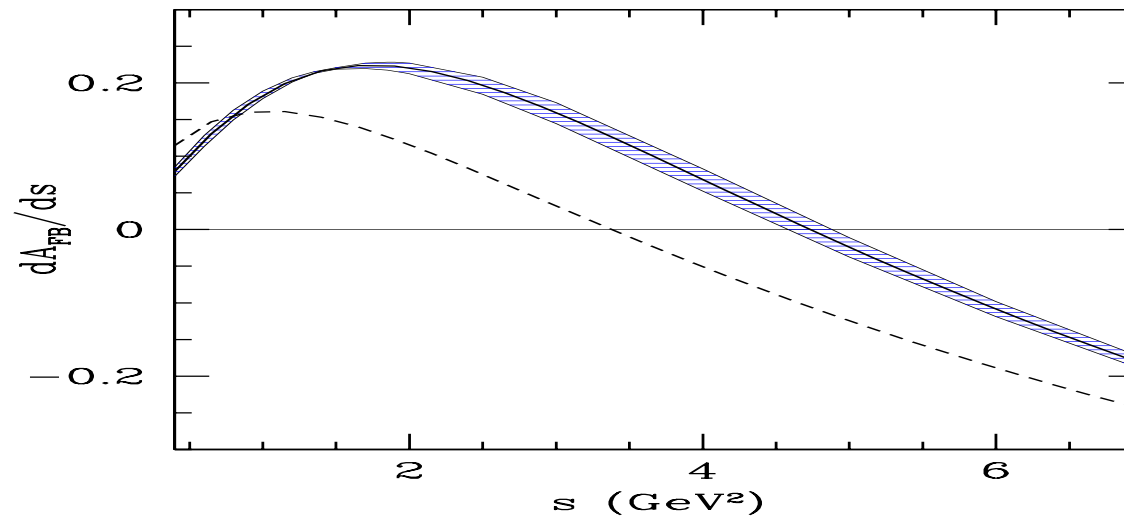
## $O(\alpha_s)$ corrections to FB-Asymmetry in $B \rightarrow K^* \ell^+ \ell^-$

- $O(\alpha_s)$  corrections to the LEET-symmetry relations lead to substantial perturbative shift in  $\hat{s}_0$  [Beneke, Feldmann, Seidel '01]

$$C_9^{eff}(\hat{s}_0) = -\frac{2m_b M_B}{s_0} C_7^{eff} \left(1 + \frac{\alpha_s C_F}{4\pi} \left[\ln \frac{m_b^2}{\mu^2} - L\right]\right) + \frac{\alpha_s C_F}{4\pi} \frac{\Delta F_\perp}{\xi_\perp(s_0)}$$

[AA, A.S. Safir (hep-ph/02054)]

H



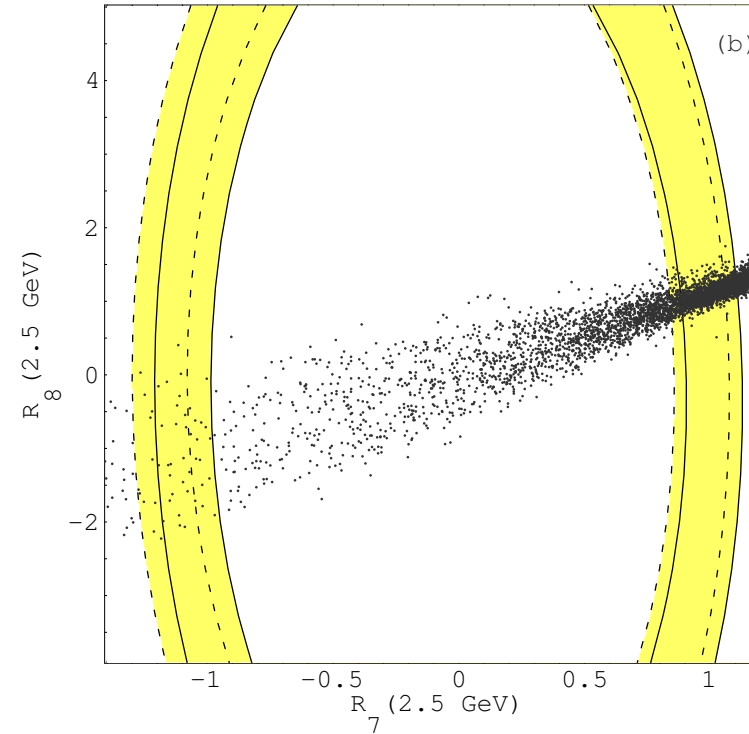
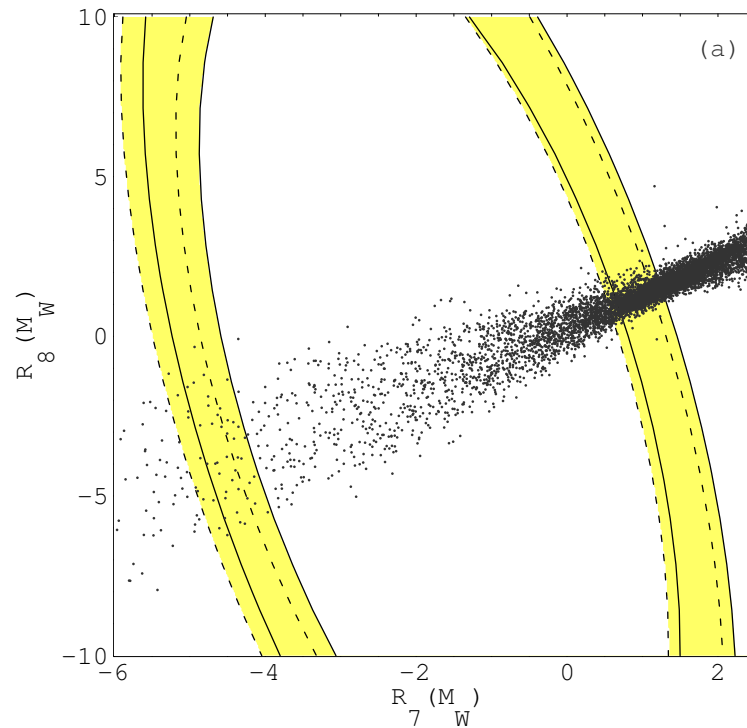
Forward-backward asymmetry  $dA_{FB}(B \rightarrow K^* \ell^+ \ell^-)/ds$  at next-to-leading order (solid center line) and leading order (dashed)

## A Model-independent Analysis of $B \rightarrow X_s \gamma$ & $B \rightarrow X_s \ell^+ \ell^-$

- Assume  $\mathcal{H}_{eff}^{SM}$  a sufficient operator basis also for Beyond-the-SM physics
- Shifts due to Beyond-the-SM [BSM] physics only in  $C_7(\mu_W), C_8(\mu_W), C_9(\mu_W),$  and  $C_{10}(\mu_W)$
- BSM Coefficients:  $R_7 - 1, R_8 - 1, C_9^{NP},$  &  $C_{10}^{NP}$
- Define:  $R_{7,8}(\mu_W) \equiv \frac{C_{7,8}^{tot}(\mu_W)}{C_{7,8}^{SM}(\mu_W)}$   
with  $C_{7,8}^{tot}(\mu_W) = C_{7,8}^{SM}(\mu_W) + C_{7,8}^{NP}(\mu_W)$
- Set the scale  $\mu_W = M_W,$  and use RGE to evolve  
$$R_{7,8}(\mu_W) \rightarrow R_{7,8}(\mu_b) = \frac{A_{7,8}^{tot}(\mu_b)}{A_{7,8}^{SM}(\mu_b)}$$
- Impose constraints from  $R_7(\mu_b)$  and  $R_8(\mu_b)$  from  $B \rightarrow X_s \gamma$  Data
- Use Data on  $B \rightarrow (X_s, K^*, K) \ell^+ \ell^-$  BRs to constrain  $C_9^{NP}$  and  $C_{10}^{NP}$
- Two-fold ambiguity due to the sign of  $C_7^{eff}$  can be resolved by data on  $B \rightarrow (X_s, K, K^*) \ell^+ \ell^-$

## Simulation of $B \rightarrow X_s \gamma$ in SUSY-MFV Models

- 90% C.L. bounds in the  $[R_7(\mu), R_8(\mu)]$  plane from the  $\mathcal{B}(B \rightarrow X_s \gamma)$   
 $\mu = m_W$  (left-hand plot);  $\mu = 2.5$  GeV (right-hand plot)



$$A_7^{\text{tot}} - \text{negative} : -0.37 \leq A_7^{\text{tot}, < 0}(2.5 \text{ GeV}) \leq -0.17$$

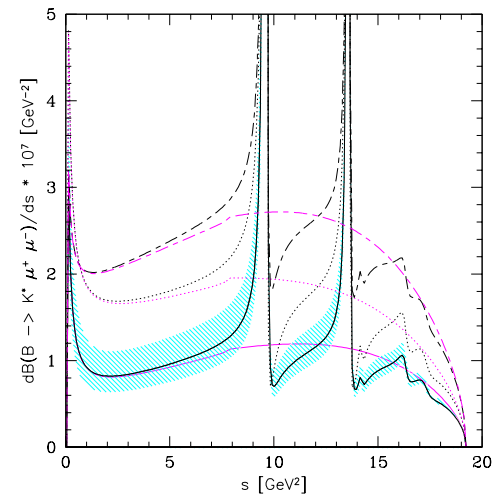
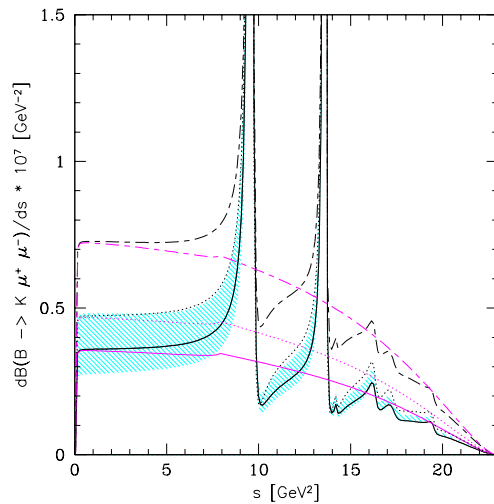
$$A_7^{\text{tot}} - \text{positive} : 0.21 \leq A_7^{\text{tot}, > 0}(2.5 \text{ GeV}) \leq 0.43$$

# Dilepton mass-Spectrum in $\bar{B} \rightarrow (K, K^*)\ell^+\ell^-$ in SM and SUSY AA, Ball, Handoko, Hiller; hep-ph/9910221

- NP contributions coded in  $R_i(\mu)$ ;  $i = 7, 9, 10$

$$R_i(\mu) \equiv \frac{C_i^{\text{NP}} + C_i^{\text{SM}}}{C_i^{\text{SM}}}$$

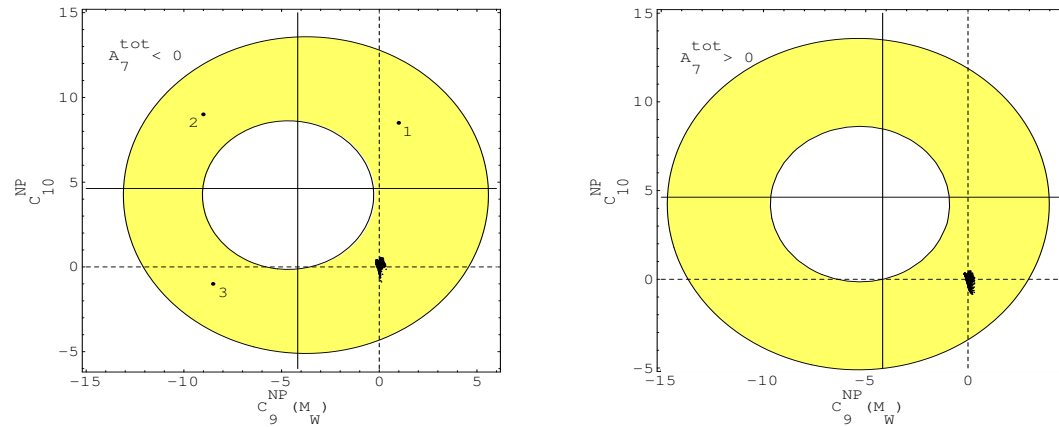
- SM (solid); SUGRA [ $R_7 = -1.2$ ] (dots);
- MIA [ $R_7 = -0.83, R_9 = 0.92, R_{10} = 1.6$ ] (dashed)



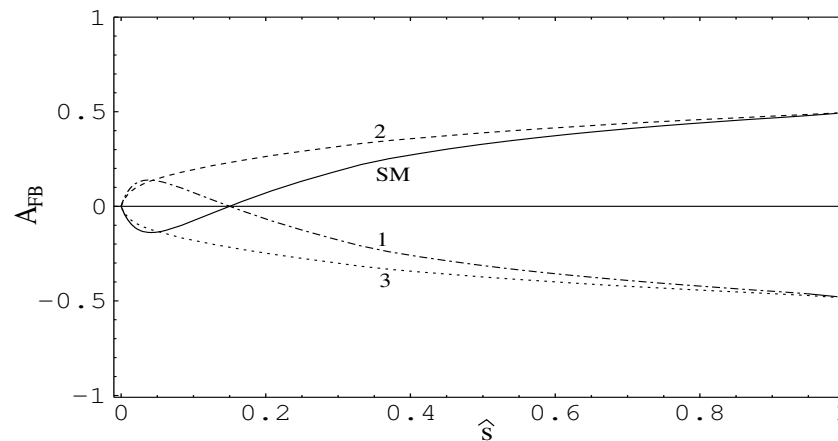
# Combined analysis of $B \rightarrow X_s \gamma$ & $B \rightarrow (X_s, K, K^*) \ell^+ \ell^-$

[ A.A., Lunghi, Greub, Hiller; DESY 01-217; hep-ph/0112300]

- Constraints from radiative and semileptonic rare decays (Points: SUSY-MFV Model)



- FB asymmetry for  $\bar{B} \rightarrow X_s \ell^+ \ell^-$ , corresponding to the points indicated above



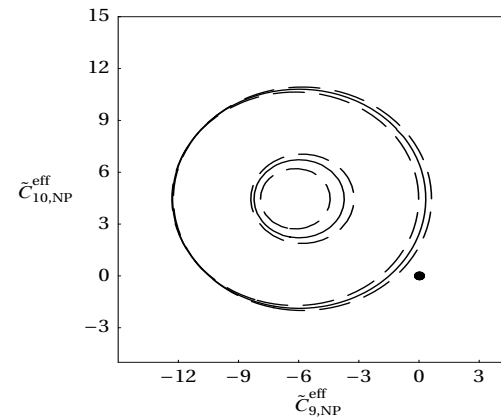
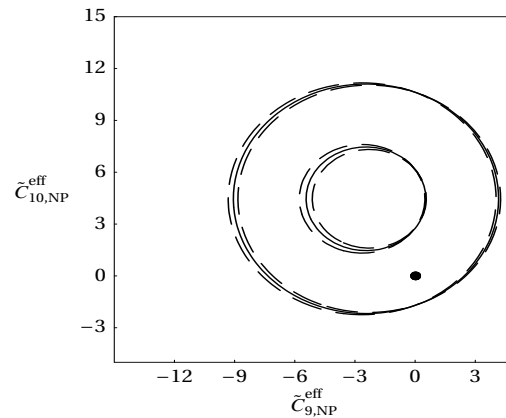
## First hints on the sign of the $B \rightarrow X_s \gamma$ amplitude

[Gambino, Haisch, Misiak; hep-ph/0410155]

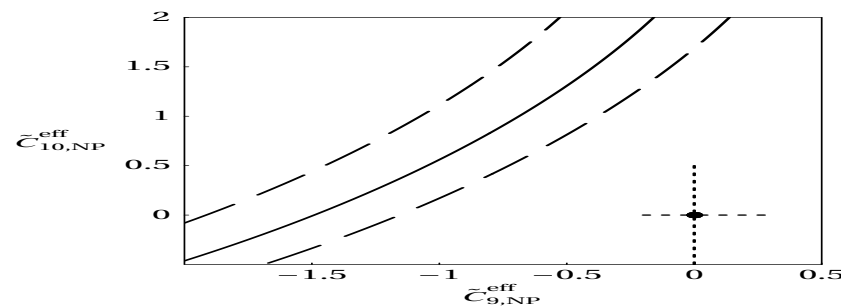
90% C.L. constraints from  $B \rightarrow X_s \gamma$  and  $B \rightarrow X_s \ell^+ \ell^-$

$C_7$  SM-like (left frame)

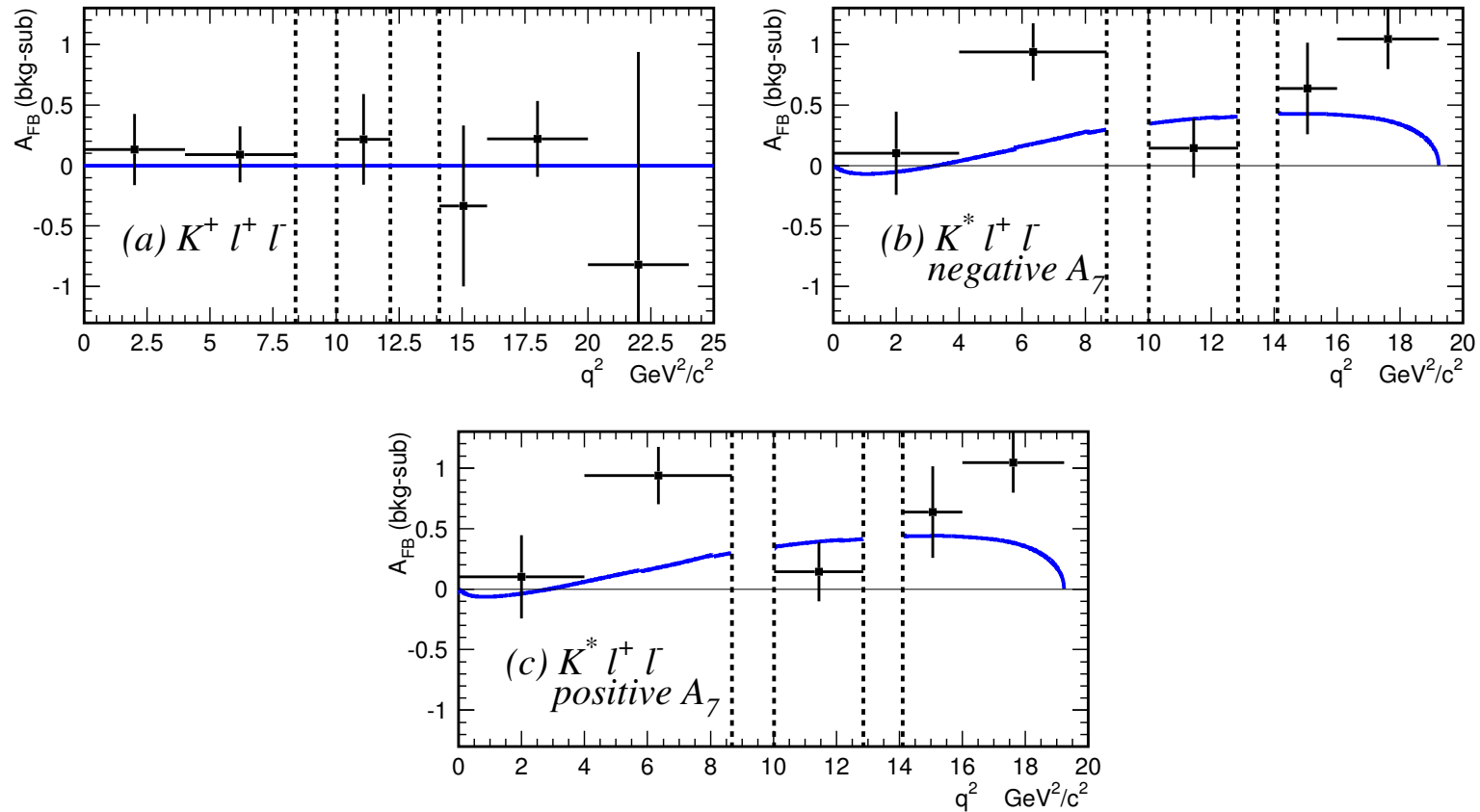
$C_7$  opposite sign (right frame)



Surroundings of the origin in the right frame above; dashed lines: MFV-MSSM



## Belle FB Asymmetry Distributions (EPS 2005)



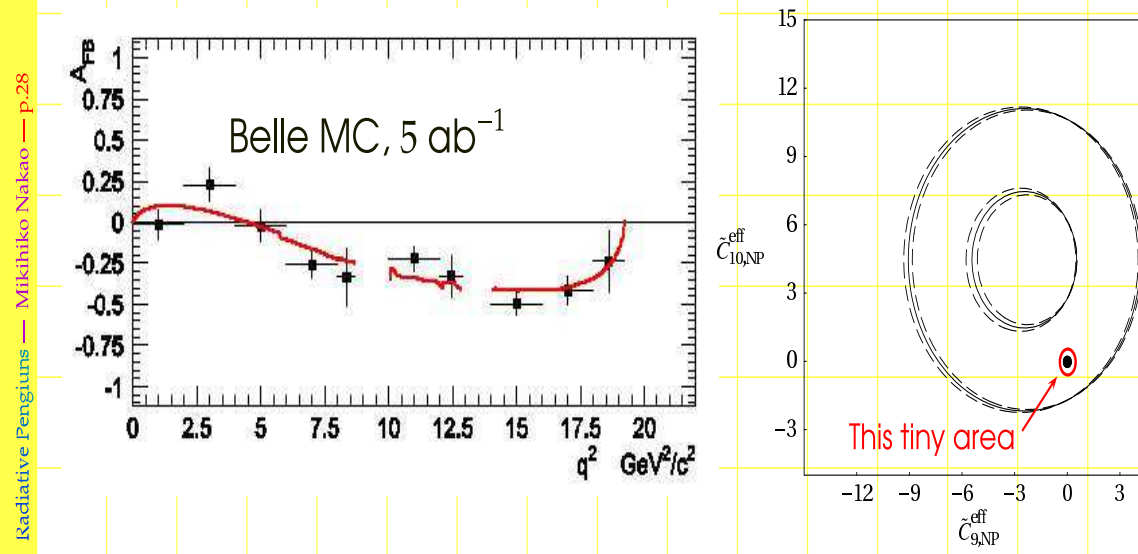
### Best Fits

- $A_7 = -0.33$ :  $A_9/A_7 = -15.3_{-4.8}^{+3.4}$ ;  $A_{10}/A_7 = 10.3_{-3.5}^{+5.2}$
- $A_7 = +0.33$ :  $A_9/A_7 = -16.3_{-5.7}^{+3.7}$ ;  $A_{10}/A_7 = 11.1_{-3.9}^{+6.0}$
- SM:  $A_7 = -0.33$ ;  $A_9/A_7 = -12.3$ ;  $A_{10}/A_7 = 12.8$

# Prospects of precise determination of $C_9$ , $C_{10}$ at Super-B Factory

## Extracting $C_9$ and $C_{10}$ from $B \rightarrow K^* \ell^+ \ell^-$

- Precise determination of  $C_9$  and  $C_{10}$  is possible
- $\Delta C_9/C_9 \sim 11\%$ ,  $\Delta C_{10}/C_{10} \sim 13\%$  at  $5 \text{ ab}^{-1}$ ,  $C_7$  fixed from  $b \rightarrow s \gamma$ 
  - Current branching fraction / background extrapolated
  - Fit to 2-dim  $q^2$  vs angular distribution, not simple  $A_{FB}$
  - Systematic error is neglected



## LHC-B MC Studies

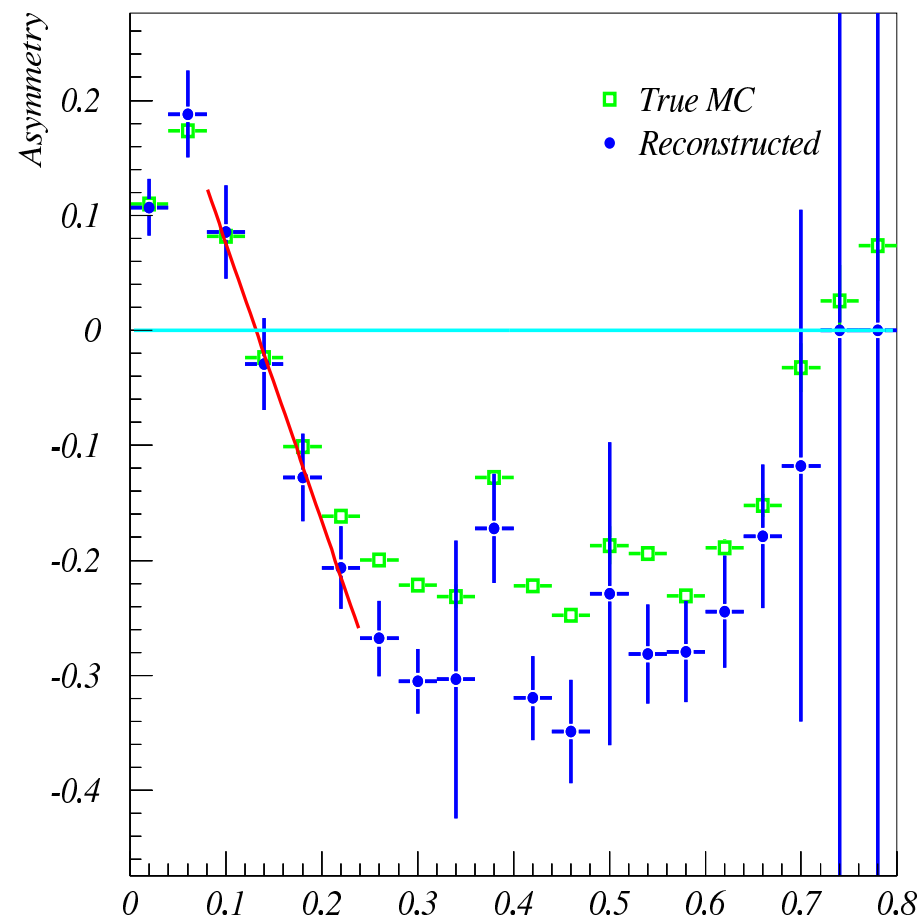


Figure 4: FB Asymmetry versus  $\hat{s}$  for  $B \xrightarrow{\hat{s}} \mu^+ \mu^- K^*$  (from Koppenburg)

## $B_s \rightarrow \mu^+ \mu^-$ in SM

- Effective Hamiltonian

$$\mathcal{H}_{eff} = -\frac{G_F \alpha}{\sqrt{2}\pi} V_{ts}^* V_{tb} \sum_i [C_i(\mu) \mathcal{O}_i(\mu) + C'_i(\mu) \mathcal{O}'_i(\mu)]$$

$$\begin{aligned} \mathcal{O}_{10} &= (\bar{s}_\alpha \gamma^\mu P_L b_\alpha) (\bar{l} \gamma_\mu \gamma_5 l), & \mathcal{O}'_{10} &= (\bar{s}_\alpha \gamma^\mu P_R b_\alpha) (\bar{l} \gamma_\mu \gamma_5 l) \\ \mathcal{O}_S &= m_b (\bar{s}_\alpha P_R b_\alpha) (\bar{l} l), & \mathcal{O}'_S &= m_s (\bar{s}_\alpha P_L b_\alpha) (\bar{l} l) \\ \mathcal{O}_P &= m_b (\bar{s}_\alpha P_R b_\alpha) (\bar{l} \gamma_5 l), & \mathcal{O}'_P &= m_s (\bar{s}_\alpha P_L b_\alpha) (\bar{l} \gamma_5 l) \end{aligned}$$

$$\begin{aligned} \text{BR}(\bar{B}_s \rightarrow \mu^+ \mu^-) &= \frac{G_F^2 \alpha^2 m_{B_s}^2 f_{B_s}^2 \tau_{B_s}}{64\pi^3} |V_{ts}^* V_{tb}|^2 \sqrt{1 - 4\hat{m}_\mu^2} \\ &\times \left[ \left(1 - 4\hat{m}_\mu^2\right) |F_S|^2 + |F_P + 2\hat{m}_\mu^2 F_{10}|^2 \right] \end{aligned}$$

where  $\hat{m}_\mu = m_\mu/m_{B_s}$  and

$$F_{S,P} = m_{B_s} \left[ \frac{C_{S,P} m_b - C'_{S,P} m_s}{m_b + m_s} \right], \quad F_{10} = C_{10} - C'_{10}$$

$$\text{BR}(\bar{B}_s \rightarrow \mu^+ \mu^-)_{\text{SM}} = (3.46 \pm 1.5) \times 10^{-9} \quad [\text{Buchalla, Buras}]$$

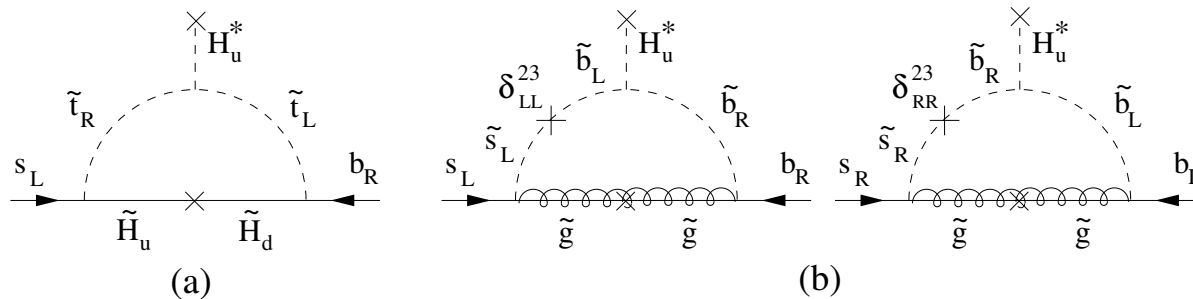
$$f_{B_s} = (230 \pm 30) \text{ MeV}$$

## $B_s \rightarrow \mu^+ \mu^-$ in Supersymmetric Models

- The decay  $B_s \rightarrow \mu^+ \mu^-$  probes essentially the Higgs sector of Supersymmetry, a type-II two-Higgs doublet model; One Higgs field ( $H_u$ ) couples to the up-type quarks, the other ( $H_d$ ) couples to the down-type quarks

$$\mathcal{L} = \bar{Q} Y_U U_R H_u + \bar{Q}_L Y_D D_R H_d$$

- Supersymmetry does not have discrete symmetries to protect the alignment of the Higgs boson interaction eigenbasis with the fermion mass eigenbasis; Higgs-induced FCNC interactions are generated through loops



- As  $H_u$  gets a VEV ( $v_u$ ), it contributes an off-diagonal piece to the down-type fermion mass matrix, mixing  $s_L$  and  $b_L$  by an angle  $\theta$

$$\sin \theta = y_b \epsilon v_u / m_b; \quad \text{as } m_b = y_b v_d, \quad \sin \theta = \epsilon \tan \beta$$

- $\mathcal{A}(b\bar{s} \rightarrow \mu^+ \mu^-) \simeq \sin \theta \mathcal{A}(b\bar{b} \rightarrow \mu^+ \mu^-) \propto \tan \beta / \cos^2 \beta \implies \tan^3 \beta$  for large- $\tan \beta$

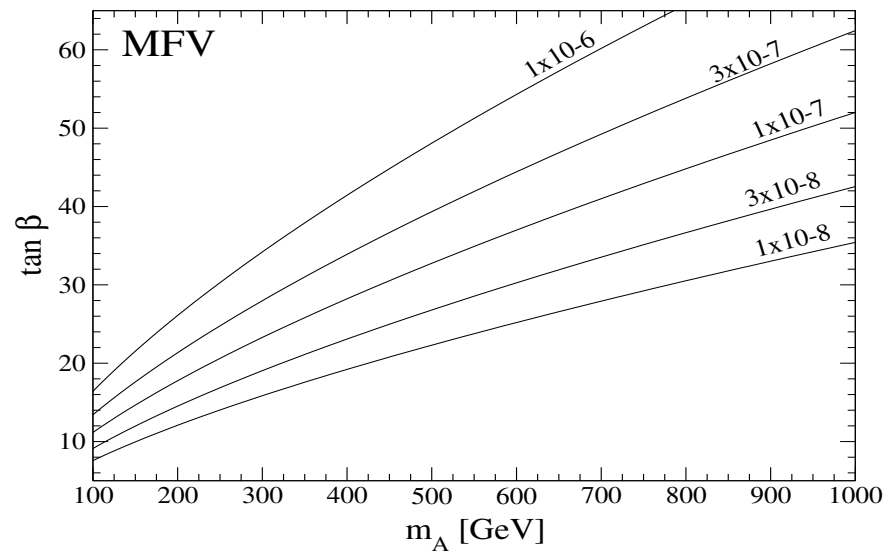
## $B_s \rightarrow \mu^+ \mu^-$ in Minimal Flavor Violation SUSY Models

- Higgsino contribution to  $\mathcal{B}(B_s \rightarrow \mu^+ \mu^-)$  [Babu, Kolda;...]

$$\mathcal{B}(B_s \rightarrow \mu\mu) \simeq \frac{G_F^2}{8\pi} \eta_{\text{QCD}}^2 m_{B_s}^3 f_{B_s}^2 \tau_{B_s} m_b^2 m_\mu^2 \left( \frac{\tan^2 \beta}{\cos^4 \beta} \right) \left( \frac{\kappa_{\tilde{H}}^2}{m_A^4} \right).$$

- $\eta_{\text{QCD}} \simeq 1.5$  is the QCD correction due to the RG between the SUSY and  $B_s$  scales

$$\kappa_{\tilde{H}} = -\frac{G_F m_t^2 V_{ts} V_{tb}}{4\sqrt{2}\pi^2 \sin^2 \beta} \mu A_t f(\mu^2, m_{\tilde{t}_L}^2, m_{\tilde{t}_R}^2)$$



## Constraints from $\mathcal{B}(B_s \rightarrow \mu^+ \mu^-)$ on SUSY Models

- **CDF  $B_s^0/B_d^0 \rightarrow \mu^+ \mu^-$  Limits [hep-ex/0508036]:**

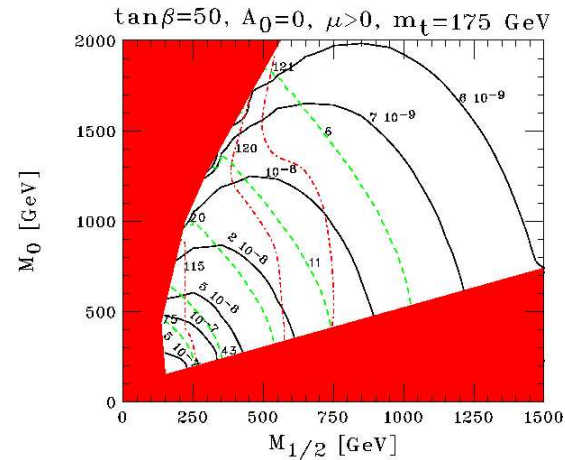
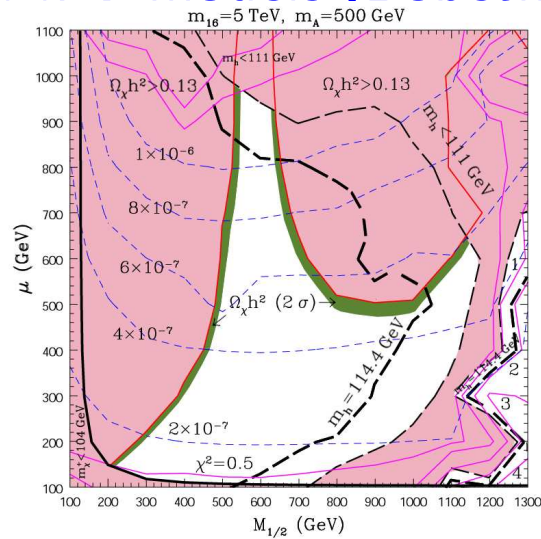
$$\mathcal{B}(B_s \rightarrow \mu^+ \mu^-) < 1.5 \times 10^{-7} (2.0 \times 10^{-7}) \text{ at } 90\% (95\%) \text{ CL}$$

$$\mathcal{B}(B_d \rightarrow \mu^+ \mu^-) < 3.9 \times 10^{-8} (5.1 \times 10^{-8}) \text{ at } 90\% (95\%) \text{ CL}$$

- **D0  $B_s^0 \rightarrow \mu^+ \mu^-$  Limits [D0note 4733-Conf (2005)]:**

$$\mathcal{B}(B_s \rightarrow \mu^+ \mu^-) < 3.2 \times 10^{-7} (4.0 \times 10^{-7}) \text{ at } 90\% (95\%) \text{ CL}$$

$\implies$  complementary limits on models of BSM physics, such as MSUGRA [Dedes et al., hep-ph/0108037], SO(10) [Dermisek et al., hep-ph/0304101; Foster et al., hep-ph/0506146] and MFV models [Bobeth et al., hep-ph/0505110]



## Summary

- All current measurements involving CP violation and FCNC processes (decay rates and distributions) are in agreement with the SM expectations
- A non-trivial test of the CKM paradigm for CP violation in the  $K$ - and  $B$ -meson sectors has been carried out at the current  $B$ -factories by overconstraining the CKM unitarity triangle
- $B$ -factories have measured all three inner angles of the UT triangle:  
 $\alpha = (99_{-9}^{+12})^\circ$ ;  $\beta = (21.7 \pm 1.3)^\circ$ ;  $\gamma = (66 \pm 17)^\circ$
- Largest current discrepancy from SM is in CPV  $b \rightarrow s\bar{s}s$  penguins;  $3\sigma$  effect
- Rare  $B$ -decays and  $B^0 - \bar{B}^0$  mixings have made a great impact on the determination of the CKM matrix elements in the third row of  $V_{\text{CKM}}$ ; In particular  
 $B \rightarrow X_s \gamma \implies V_{ts} = -(46.0 \pm 8.0) \times 10^{-3}$   
 $B \rightarrow (\rho, \omega, K^*) \gamma \implies \left| \frac{V_{td}}{V_{ts}} \right| = 0.200_{-0.025}^{+0.026} \quad {}_{-0.029}^{+0.038}$
- A number of benchmark measurements remain to be done. These include, among others,  $\mathcal{B}(B_s \rightarrow \mu^+ \mu^-)$  and  $\Delta M_{B_s}$ , which will be carried out at Fermilab and LHC; Correlations between these and other Rare  $B/K$ -decays crucial to disentangle BSM physics in the flavour sector
- Hope that the synergy of high energy frontier and low energy precision physics, which worked so well in piecing together the SM, will continue to hold sway in the LHC-era, providing valuable information about the flavour aspects of the BSM physics